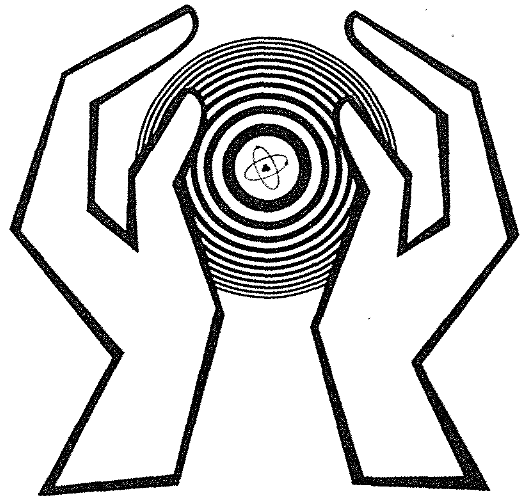
R. Hazeltine

CONF-740403



Proceedings of annual meeting on

Theoretical aspects of Controlled Thermonuclear Research

Held at Lawrence Hall of Science
University of California - Berkeley, California

April 3-5, 1974 Sponsored by

Lawrence Livermore Laboratory

Prepared for U.S. Atomic Energy Commission under contract No. W-7405-Eng-48

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TID-4500, UC-20
Controlled Thermonuclear
Processes

LAWRENCE LIVERMORE LABORATORY
University of California/Livermore, California

PROCEEDINGS OF THE ANNUAL MEETING ON THEORETICAL ASPECTS OF CONTROLLED THERMONUCLEAR RESEARCH

Held at

LAWRENCE HALL OF SCIENCE

University of California/Berkeley, California

April 3— 5, 1974

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SCHEDULE

1974 ANNUAL MEETING ON THEORETICAL ASPECTS OF CTR

TUESDAY, 2 April

Registration at Claremont Hotel (7:00 PM – 9:00 PM)

WEDNESDAY, 3 April

9:00 AM	Morning Sessions Begins - Auditorium, Lawrence Hall of Science
10:45 AM	Coffee Break
11:00 AM	Meeting Reconvenes
1:00 PM	Lunch - Dining Room, Lawrence Hall of Science
2:00 PM	Afternoon Session Begins
3:45 PM	Coffee Break
4:00 PM	Special Session - Trapped Particle Modes
5:30 PM	Afternoon Session Adjoins

THURSDAY, 4 April

9:00 AM	Morning Session Begins - Auditorium, Lawrence Hall of Science
10:45 AM	Coffee Break
11:00 AM	Meeting Reconvenes
1:00 PM	Lunch - Dining Room, Lawrence Hall of Science
2:00 PM	Afternoon Session Begins
3:45 PM	Coffee Break
4:00 PM	Special Session - Two Component Systems
5:30 PM	Afternoon Session Adjoins
6:00 PM	Social Hour - Dining Room, Men's Faculty Club, Campus
7:30 PM	Dinner - Men's Faculty Club, Campus

FRIDAY, 5 April

9:00 AM	Morning Session Begins
10:45 AM	Coffee Break
11:00 AM	Meeting Reconvenes
1:00 PM	Lunch - Dining Room, Lawrence Hall of Science
2:00 PM	Meeting Reconvenes
3:45 PM	Coffee Break
4:00 PM	Special Session - Pinches
5:30 PM	Meeting Adjoins

PROGRAM

1974 ANNUAL MEETING ON THEORETICAL ASPECTS OF CTR

WEDNESDAY, 3 April Session A Chairman: H. Grad

A1 9:00 AM J. P. Freidberg
Scyllac, Theory and Experiment.

A2 9:50 AM W. M. Tang
Trapped Particle Modes in Tokamak.

A3 H. Weitzner
Guiding Center Plasma Effects in Helically Symmetric Equilibria

A4 M. Z. Caponi, R. C. Davidson, S. Hamasaki, N. A. Krall, P. C. Liewer, D. B. Batchelor, D. D'Ippolito and J. M. Ogden
Kinetic and Fluid-Numerical Studies of High-β Plasmas.

A5 D. A. Baker and J. A. Phillips
Energy and Pressure Balance for a Plasma Column Energized with a Pulsed Longitudinal Current.

A6 C. K. Chu, H. C. Lui, A. M. M. Todd
Two-Dimensional Numerical Simulation of High-Beta Pinches and Toroids.

A7 C. L. Hedrick, G. E. Guest and D. B. Nelson
Vlasov Treatment of Ballooning Modes in Hot Electron Plasmas.

SESSION B 2:00 PM Chairman: H. H. Fleischmann

B1 W. A. Newcomb
Focal-Point Instabilities: An Apparent Counter-example to the MHD Energy Principle.

B2 H. Grad
Adiabatic Evolution of Islands.

B3 M. S. Chance, R. C. Grimm, J. M. Greene, J. L. Johnson, and K. E. Weimer
Numerical Determination of Mode Spectra in a Finite Straight Tokamak with Uniform Axial Current and Elliptic Cross Section.

B4 B. K. Harrison
The Topolotron: A New Concept in Magnetic Confinement.

Weds. Cont'd

SPECIAL SESSION 4:00 PM TRAPPED PARTICLES Chairman: M. N. Rosenbluth

C. O. Beasley, J. D. Callen, H. R. Hicks and J. F. Seely
Shear Stabilization Criterion for the Dissipative Trapped-Electron Instability.

C. S. Liu and M. N. Rosenbluth
Dissipative Universal Instability Due to Trapped Electrons in Toroidal Systems and Anomalous Diffusion.

W. Horton, Jr. and D. W. Ross
Radial Normal Mode Stability Theory of Trapped Electron Mode.

M. Dobrowolny
Stabilization of the Trapped-Ion Dissipative Instability by Impurities.

B. Coppi
Anomalous Resistivity and Transport Due to Impurities in High Temperature Plasmas.

D. K. Bhadra
Trapped Particle Modes in the Presence of Impurities.

H. L. Berk and M. N. Rosenbluth
Ion Cyclotron Instability Resulting from Ion-Trapped Particle Mode.

S. K. Wong, D. K. Bhadra, D. Dobrott
Effect of Collisions on Impurity Drift Waves.

THURSDAY, 4 April SESSION C Chairman: C. W. Horton, Jr.

C1 9:00 AM P. H. Rutherford
MHD Oscillations in Tokamaks, Theory and Experiment.

C2 9:50 AM J. D. Callen
Anomalous Resistivity in Tokamaks?

C3 M. N. Rosenbluth, R. White, D. Monticello and H. Strauss
Non-Linear Kink and Tearing Modes in Tokamaks.

R. White, D. Monticello, M. N. Rosenbluth and H. Strauss
Numerical Studies of Non-Linear Evolution of Kink Modes in Tokamaks.

C4 J. U. Brackbill
Numerical Studies in Non-Linear Magnetohydrodynamics.

C5 D. Potter and G. Tuttle
A Waterbag Simulation of the Tokamak.

Thurs. Cont'd

C6 R. D. Hazeltine and F. L. Hinton
Kinetic Theory of Plasma Scrape-Off in a Divertor Tokamak.

C7 J. A. Rome, D. G. McAlees, J. D. Callen and R. H. Fowler
Particle Orbit Loss-Cones in Tokamaks.

SESSION D 2:00 PM Chairman: C. S. Liu

D1 A. A. Ware
The Importance of Electrostatic Trapping of High-Z Impurity Ions in Tokamak Plasmas.

D2 D. Dobrott, M. S. Chu and S. K. Wong
Interchange Instability in Non-Circular Cross-Section Models.

D3 F. W. Perkins, F. Troyon and J. L. Sperling
On Radio Frequency Heating of Tokamaks.

D4 M. N. Rosenbluth, H. L. Berk, P. H. Rutherford, N. R. Sauthoff and C. S. Liu
Stability of an Ion Beam to Velocity Space Instabilities.

SPECIAL SESSION 4:00 PM TWO-COMPONENT SYSTEMS Chairman: H. P. Furth

K. Papadopoulos and J. Boris
On the Role of Suprathermal Fluctuations on the Energetic Ions in a Two-Component Wet Wood Burner Plasma.

D. L. Jassby and H. P. Furth
Two-Component Fusion Reactors with Sustained Suprathermal-Ion Energy.

B. N. A. Lamborn
The Ion Two-Stream Instability.

N. Winsor, D. Anderson, C. Wagner and J. Boris
Studies of the Wet Wood Burner.

J. Killeen, T. Johnson, A. A. Mirin and M. E. Rensink
Computational Studies of the Two-Component Torus.

W. Heckrotte and J. R. Hiskes
Parametric Survey for the Linear Two-Component System.

J. A. Byers, J. Killeen, B. McNamara and A. A. Mirin
Computational Studies of Two Component Mirror Reactor Concepts.

FRIDAY, 5 April SESSION E Chairman: John Dawson

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E2 9:50 AM N. Rostoker
E-Beam Heating, Theory and Experiment.

E3 D. E. Baldwin
Suggested Role of Electron Cyclotron Noise in Stabilizing the Drift-Cone Mode in Mirror Machines.

E4 R. E. Aamodt
Particle Losses Induced by Highly Localized Flute Modes.

E5 T. Hatori and T. Watanabe
Conceptual Design of a Controlled Thermonuclear Reactor with the Adiabatic RF Plugging Scheme.

E6 J. R. Freeman, J. W. Poukey, M. M. Widner, M. J. Clauser, T. P. Wright and G. Yonas
Theoretical Problems in Electron Beam Fusion.

E7 G. Schmidt
Inhomogeneous Threshold in Beam-Plasma Systems.

SESSION F 2:00 PM Chairman: Laurence S. Hall

F1 H. Okuda
Large Amplitude Convective Cells and the Low-Frequency Electrostatic Instabilities.

T. Kamimura and J. M. Dawson
Effect of Mirroring on Convective Diffusion of Plasma.

F2 J. A. Krommes and C. R. Oberman
Fluctuations and Cross-Field Transport in Guiding Center Plasma.

J. M. Dawson
Plasma Diffusion for a Two-Dimensional Model.

F3 T. M. O'Neil
Probability Distributions for the Electric Field in Weak Turbulence Theory.

F4 F. Winterberg
Ignition of Thermonuclear Micro-Explosions with Intense Ion Beams.

SPECIAL SESSION 4:00 PM PINCHES Chairman: Harold Weitzner

G. E. Cooper and C. W. Hartman
2-D MHD Z-Pinch Calculations.

Y. Pao
Guiding Center Screw Pinch Spectrum and Stability.

G. Vahala
Spectra and Growth Rates of a Generalized Screw Pinch.

W. Grossmann
High Beta Toroidal Belt Pinch Stability.

A. Kadish and D. Stevens
Belt Pinch Free Boundary Equilibria.

A. Kadish
Equilibrium Theory - Toroidal Belt in a Force Free Field.

S. Ortolani
Numerical and Theoretical Investigations on a Reverse Field Pinch Experiment at Padua University.

A. L. Hoffman, G. C. Vlases, H. Rutkowski, H. Hoida
Theory and Experiment on Laser Heated Pinches and Solenoids.

CONTENTS

Particle Losses Induced by
Highly Localized Flute Modes

by

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Washington, D.C. 20375

and

University of Colorado*
Boulder, Colorado 80302

Some of the high frequency flute-like modes common to open-ended containment devices are highly localized in radial extent. The non-linear saturation and induced particle loss rates of such modes are much different than the equivalent homogeneous plasma mechanisms. These differences are derived and discussed in detail for the localized Drift-Cone mode and shown to imply containment times, orders of magnitude longer than previous estimates. The implications of the localized model are compared to contemporary mirror containment results.

*Research supported in part by AEC contract #AT(11-1)-2081 with the University of Colorado

ENERGY AND PRESSURE BALANCE FOR A PLASMA COLUMN ENERGIZED
WITH A PULSED LONGITUDINAL CURRENT

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Los Alamos, New Mexico 87544

In fusion research there is revived interest in z pinches having magnetic field configurations with diffuse profiles since stable configurations exist which confine high beta plasmas according to MHD theory.[1,2] Experimentally the poloidal field diffuses rapidly and most of the profiles now being considered have longitudinal currents nearly uniform inside the pinched plasma column. In many experiments the plasma β_θ's are high. The production of stable configurations would be greatly simplified by operating at lower values which by MHD theory would have a greater margin of stability.

These considerations suggest that there are limitations on β_θ due to the processes involved in setting up the plasma and field profiles. In particular, the plasma is ohmically heated during the diffusion of the poloidal field.

This paper treats this problem using general energy and pressure balance conditions for a mixed-field plasma column. Conditions of global energy and pressure balance are applied to a cylindrically symmetric plasma column having a B_θ confining field and a B_z stabilizing field as exist in stabilized z pinches and tokamaks. We have shown under general assumptions which are independent of the mechanisms of field diffusion, that without losses the pressure as the poloidal field diffuses and the longitudinal current becomes uniform, can grow too large for the confining field to contain.[3] Necessary conditions to obtain equilibrium with negligible losses have been derived.

*Work performed under the auspices of the United States Atomic Energy Commission.

References

[1] D. C. Robinson, Plasma Physics 13, 439 (1971).

[2] D. A. Baker, et. al., Plasma Physics and Controlled Nuclear Fusion Research.

[3] D. A. Baker and J. A. Phillips, Phys. Rev. Lett. 32, 202 (1974).

SUGGESTED ROLE OF ELECTRON CYCLOTRON NOISE IN STABILIZING THE DRIFT-CONE MODE IN MIRROR MACHINES*

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Livermore, California 94550 USA

ABSTRACT

Cross field electron diffusion can be shown to stabilize the drift cone mode only if the diffusion coefficient

$$D_e \gtrsim \omega_{ci}\, a_i^3 T_e / T_i R_p$$

where R_p is the plasma radius. However, because the ion velocity scattering rate is known from the plasma lifetime, this level of electron diffusion can be achieved only by a high frequency instability which is ineffective at ion scattering.

It is shown that the lower branch ($\omega<\omega_{ce}$) of electron plasma oscillations in a high density ($\omega_{pe} \sim \omega_{ce}$) mirror machine are unstable due to the high parallel velocity electron loss hyperbola. For mirror ratio 2 and ambipolar potential $3T_e$, the growth rate $\gamma \approx 5.10^{-4}\, \omega_{ce}$, and wave numbers $ka_e \approx 0.1$. Estimating the level of electron diffusion as that necessary to stabilize this mode, $D_e = \mathrm{Max}[\gamma/k^2]$, predicts stability for the drift cone mode if

$$\frac{\omega_{pe}}{\omega_{ce}} \gtrsim 2\,\frac{a_i}{R_p}$$

(for 2x II parameters). More detailed comparison of the predictions of this theory with experimental experience will be described.

*Work performed under the auspices of the U.S. Atomic Energy Commission

STABILITY OF A ROTATING CONDUCTING LINER

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We have considered the stabilizing effect of liner rotation in connection with two theoretical models, for the theta-pinch and Flying Cusp geometries respectively. The former model assumes an inviscid incompressible liner, perfectly electrically conducting and exhibiting zero heat conductivity and with no tendency to vaporize or ionize. The liner has the form of cylindrical shell which thickens during the course of the implosion driven by the resultant of the inside and outside magnetic pressures. At $t = 0$, an irrotational perturbation is imposed. The linearized perturbation equations are solved by numerical integration. The resulting surface displacements are used to quantify the tendency of the liner to disrupt or to encroach into the confined plasma. Inner interface perturbations grow sharply after turnaround. If sufficient rotational velocity is present, centrifugal acceleration stabilizes the inner surface. We present the results of a parameter study, showing the effect of various amounts of rotation on liner stability for different flute modes m, $1 < m < 100$. The results are displayed in the form of stability envelopes for the perturbations. We demonstrate that increasing liner thickness tends to stabilize against disruption.

Some conclusions can be drawn from these results which are also applicable to the Flying Cusp geometry. In particular, the increasing stability against disruption with increasing thickness, and the tendency of rotation to stabilize against encroachment toward the axis still hold. To study certain other modes, a second model is considered. Two circular filaments of small aspect ratio, not necessarily coaxial, implode under the influence of magnetic fields produced by currents flowing around their major circumferences. The results of these studies indicate that rotation does not stabilize the scallop modes, but can reduce growth rates if the two rings rotate oppositely. If they rotate in the same sense, the instability is much more dangerous.

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SHEAR STABILIZATION CRITERION FOR THE DISSIPATIVE TRAPPED-ELECTRON INSTABILITY*

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Of the various types of low frequency drift and trapped-particle instabilities,[1] the most relevant mode in present tokamaks is the drift-dissipative trapped-electron instability. This mode is basically a drift wave destabilized by the dissipation due to collisions of trapped electrons with untrapped electrons. Magnetic shear has the effect of localizing this mode near a mode-rational surface and of convecting wave energy away from the locally unstable region, which has a damping effect. The instability can be stabilized if the shear is strong enough so that the energy convection damping exceeds the collision-induced growth.[2] In order to precisely determine the amount of shear required, the collision-induced growth must be accurately known. In previous work the growth has been estimated with a Krook model collision term,[3] or with a Fokker-Planck collision operator in the limit of high,[1,2] or low[1] collision frequency. In this work the "model operator" employed in neoclassical transport theory[4,5] is bounce-averaged and then approximated by its form for deeply trapped particles. The resultant linearized Boltzmann equation is solved for the nonadiabatic part of the perturbed distribution function by a Green's function solution of the pitch-angle diffusion problem introduced by collisions. The collisional growth term obtained by this procedure is valid for arbitrary ratios of the wave to collision frequency, but must be evaluated numerically. The shear stabilization criterion resulting from this growth term will be given and contrasted with the approximate one quoted in the literature.[2]

*Research sponsored by the U. S.A.E.C. under contract with Union Carbide Corporation.
+On leave of absence from M.I.T.
∓Permanent address: University of Tennessee, Knoxville, Tennessee. Supported by the U.S.A.E.C. under contract AT-(40-1)-2598.

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2R. Z. Sagdeev and A. A. Galeev, Comments on Plasma Physics and Controlled Fusion 1, 23 (1972).
3R. Deschamps, et al., Phys. Rev. Letters 31, 1457 (1973).
4L. M. Kovrizhnikh, Trieste Report IC-70-86, Aug., 1970.
5M. N. Rosenbluth, R. D. Hazeltine and F. L. Hinton, Phys. Fluids 15, 116 (1972).

Ion Cyclotron Instability Resulting From Ion-Trapped Particle Mode*

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ABSTRACT

The trapped-ion mode results essentially in an interchange of trapped ions, while untrapped ions and electrons remain Maxwellian. The deviation of the trapped-ion distribution from Maxwellian will give rise to an anisotropic velocity space distribution that can cause instability at the ion cyclotron frequency and its harmonics. These oscillations in turn can limit the amplitude of the low-frequency trapped-ion instability by causing an enhanced rate of scattering of the trapped ions to passing orbits. As a result, in some circumstances, the nonlinear radial diffusion due to the trapped-ion mode can be lowered from that predicted by assuming that the local density gradient needs to be destroyed.

* Work supported by U. S. AEC Contract AT(11-1)-3073.

Annual Meeting on Theoretical Aspects of CTR
LBL - Berkeley, Calif. April 3-5, 1974

Parametric Excitation of Ion Waves by Resonance-Cone Fields Near the Lower Hybrid Frequency*

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Department of Electrical Engineering
and Research Laboratory of Electronics

ABSTRACT

The excitation of ion-waves by a pump wave in the plasma near the lower-hybrid frequency has been recently described (using the symbolic computation system MACSYMA) for arbitrary propagation direction with respect to B_o.[1] However, for heating of a tokamak-type plasma the r.f. energy of the pump must originate outside the plasma, at the wall. It has also been shown recently[2] that the external, localized excitation of a plasma near the lower-hybrid frequency generates electric fields in the plasma that are of finite spatial extent along so-called resonance cones.[3] Using such a two-dimensional description of the pump field in the plasma we analyse its parametric downconversion to ion-acoustic, electrostatic ion-cyclotron, and magneto-sonic waves. We show that the major effect of the resonance-cone structure is to impose a "finite-pump-length" requirement for the interaction to be significant. For typical tokamak-type plasmas this in turn requires that the r.f. energy be supplied from a properly phased array of wave-guides at the wall. Electric field and r.f. power thresholds for heating of tokamak-type plasmas, based on this two-dimensional analysis, will be presented.

* This work was supported in part by the NSF (Grant GK-28282X1 and GK-37979X) and by the AEC (Contract AT(11-1)-3070)

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March 15, 1974

Trapped Particle Modes in the Presence of Impurities

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The nature of the trapped particle modes in a toroidal geometry has been investigated in the presence of impurity ions using a model magnetic field configuration. For opposing equilibrium density profiles for the plasma ions and impurity ions, it is found that the dissipative trapped ion mode is stabilized when the condition $n_I/n_i > (\sqrt{\epsilon}/Z)(L_n^I/L_n)$ is satisfied (n_I, n_i are the impurity and plasma ion density, respectively; ϵ = inverse aspect ratio; L_n^I, L_n are the characteristic density lengths; Z is the impurity charge number). For the dissipative temperature gradient trapped electron instability, there now exists a critical temperature gradient given by $(d \ln T/d \ln n)_{crit.} = (2/3)(n_I Z/n_o)(L_n/L_n^I)$, above which it is unstable, n_o being the total ion density. Results obtained for other instabilities will also be presented.

Work supported by the U. S. Atomic Energy Commission, Contract No. AT(04-3)-167, Project Agreement No. 38.

Large Anomalous Electron Viscosity through a Current Shear, Threshold Independent Instability

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Sheared currents parallel to the magnetic field are shown to be unstable for all shear values because of an electrostatic, purely electron mode oscillating near the cyclotron frequency. The instability gives rise to a large anomalous viscosity, so that current penetration in large tokamaks[1] can occur rapidly, i.e. on the order of a few milliseconds.

The instability is a cold electron mode that propagates at an angle between the direction of the magnetic field and the direction of the shear gradient. The mechanism for the instability is the bunching of charge along the magnetic field. The velocity of the electrons in the magnetic field direction, Z, due to the perturbed potential, ϕ, is $\delta U_z^{(a)} = (e/m)k_z\phi/\omega$. This flow of electrons tends to neutralize the plasma and thus decrease the potential. However, this effect can be opposed by the particle convection in the shear gradient direction, X. The linearized velocity in the gradient direction is $\delta U_x = (e/m)\omega k_x\phi/(\omega^2 - \Omega_e^2)$, where k_x is the effective value of the wave number, described by the eikonal approximation.

Due to the shear the convection along the gradient replaces particles with one drift velocity in the Z-direction with other particles having a different drift. The change of the velocity in the Z-direction because of this effect is of the order $\delta U_z^{(b)} \approx \delta U_x U'/\omega = (e/m)k_x\phi U'/(\omega^2 - \Omega_e^2)$, where U' is the velocity shear. Instability can occur when $|\delta U_z^{(b)}| > |\delta U_z^{(a)}|$, and this condition can be satisfied for any U' for ω sufficiently close to the cyclotron frequency.

A dispersion relation is obtained. For the case that $|| - \Omega_e^2/\omega_p^2|$ is not too small with respect to unity, we find $(\omega - \Omega_e)/\Omega_e = 1/2(k_x^2/k_x^2)/(\Omega_e^2/\omega_p^2 - 1)$ and $\gamma = k_z U'/k_x$, where $\gamma/|\omega - \Omega_e| \ll 1$.

We derive an expression for the large tokamak current penetration in terms of an anomalous kinematic viscosity. The coefficient of viscosity, μ, can be large due mainly to the factor $1/(\omega^2 - \Omega_e^2)$ in δU_x. We find

$$\mu = \frac{2\tau}{k_x^2}\Omega_e^2 \left\langle \frac{(\delta n)^2}{n^2} \right\rangle ,$$

where τ is the characteristic time interval for which the mode grows; σn is the perturbed number density. We compute the value of μ for a shear length equal to the characteristic skin depth, c/ω_p. We use the following parameters[1] in the skin region: $n = 2\cdot 10^{13}/cm^3$, $B = 50kG$. We assume that the electron temperature is 1 keV and that the minimum electron drift is $V_e/10$ to find that $\mu \approx 10^{10}$ cm^2/sec. The penetration time for a tokamak minor radius of 45 cm. is about 10^{-3}sec. Since the mode is unstable even when the shear decreases as penetration develops, we expect that the above value for μ will not decrease substantially during the process.

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Fusion Using Line Cusps

Allen H. Boozer and M. A. Levine
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Cusp geometrics are the standard examples of MHD stable, high beta systems. However, they have played only a minor role in the thermonuclear program due to theoretical predictions of a fast particle loss rate. However, recent theoretical[1] work on the straight line and the toroidal line cusp geometries indicates the particle confinement of these systems can be adequate for fusion. In this paper, the theory of particle and plasma confinement in cusps is reviewed. A method of efficiently heating cusp geometries with magneto acoustic waves is discussed, and the large advantages cusp reactors have over competing systems are considered.

1) Allen H. Boozer and M. A. Levine, Phys. Rev. Letters 31, 1287 (1973)

NUMERICAL STUDIES IN NON-LINEAR MAGNETOHYDRODYNAMICS*

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A computational algorithm for fully three-dimensional, magnetohydrodynamic flow problems has been developed for application to dynamical stability problems in a wide range of magnetic confinement geometries. The algorithm is an extension of the ICED-ALE [1,2] method, which employs an implicit difference formulation of the dynamical equations on a computation mesh constructed from arbitrarily shaped, six-surfaced cells. Such a mesh can be deformed to lie within a curvilinear boundary such as, for example, a toroidal shell. During the course of a calculation, the relative motion of the mesh and fluid is specified to reduce computational diffusion to the minimum consistent with maintaining a reasonably regular mesh. By reducing computational diffusion in this way and using a variable resolution mesh, a large class of interesting dynamical stability problems are put well within the reach of present day computers and budgets.

The algorithm has been applied to a number of problems, most recently to the internal kink mode instability in a diffuse screw pinch. When a plasma is confined within a conducting, cylindrical shell by a helical magnetic field, linear stability theory predicts that for given radial density and field profiles certain perturbations are unstable. In the cases computed, the profiles are chosen so that a perturbation of the form $\exp i(kz+\theta)$ is unstable, where k is $-2\pi/L$ and L is the length of the computation mesh. The growth of the instability is computed and exhibited by means of a number of graphical displays, including magnetic flux and isodensity contours, and vector plots.

1. C. W. Hirt, A. A. Amsden, and J. L. Cook, An Arbitrary Lagrangian-Eulerian Computing Method for All Flow Speeds, J. Comp. Phys. to be published.

2. J. U. Brackbill and W. E. Pracht, An Implicit, Almost-Lagrangian Algorithm for Magnetohydrodynamics, J. Comp. Phys. 13 (1973), 455.

*This work was performed under the auspices of the United States Atomic Enery Commission.

Closing the Credibility Gap Between Computer Models and Real CTR Devices

by O. Buneman
EE Department
Stanford University

A CTR-dedicated computer could advance, say, 10^8 particles through 1000 periods of the highest significant (gyro- or plasma-) frequency in a month. If one could believe the results, investment in such extensive runs would be justified: they can save building devices proved ineffective by simulation. Hitherto, grid instabilities (Langdon), exaggerated shot noise (Birdsall and Okuda), non-physical Cerenkov effects (Boris) and other weaknesses have discredited computer models.

To obviate these drawbacks, and to enhance awareness of what physics we are actually simulating, we are now tracing, strictly, broad (and hence quiet) superparticles with Gaussian profile (Dawson) through e-m fields consisting of "superphotons" which propagate at the correct phase velocity, and we are pushing grid effects beyond the limit of computer precision.

The equations for advancing superphotons and superparticles are obtained, following Lewis, from the action principle. A quasi-gaussian k-dependence of the electric charge serves the dual purpose of giving the superparticles their smooth profile and limiting the number of photons to be kept on record. Evaluation of the field for each particle from the "photon" representation, i.e. from the Fourier (Fourier-Fourier-Bessel for toroidal geometry) harmonics, as well as excitation of each photon by the particles, is made possible by cubic spline interpolation from a table not much larger than the number of superphotons kept (typically 10^6). To make sure that the interpolation does not introduce grid effects, a single particle was traced through the interpolation grid in 2D and its equipotentials were checked to remain co-moving circles. A movie shows this grid-insensitivity. Some 1-D runs with this model have yielded fluctuation spectra in agreement with conventional theory and with experiment.

COMPUTATIONAL STUDIES OF TWO COMPONENT MIRROR REACTOR CONCEPTS*

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ABSTRACT

Two component mirror reactor concepts and scaled laboratory experiments are examined with a variety of computational models. This work considerably extends the previously reported calculations that used a one dimensional first normal mode Fokker Planck treatment of the hot ions and held T_e, $T_{i_{cold}}$ $n_{i_{cold}}$ fixed and assumed $\emptyset = 0$. The MARTENI codes describe the dynamic equilibrium of the flowing Maxwellian target. Application to the arc in BB is described; the fraction of $I_e V$ delivered to T_e is determined with a particle simulation code. For two component models the hot injected ions are described with a two dimensional Fokker Planck code; n_{hot} (z) is calculated and with n_{cold} (z) obtained from the MARTENI code is used to generated $\emptyset(z)$. The electron and target ion parameters are not fixed but rather are dynamically calculated by the coupled MARTENI-Fokker Planck codes. In addition, self consistent magnetic field effects (adiabaticity, magnitude of self well, field reversal) are examined with an r-z particle simulation code.

*Work performed under the auspices of the U.S. Atomic Energy Commission

KINETIC AND FLUID-NUMERICAL STUDIES OF HIGH-β PLASMAS†

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The MHD equations are of questionable validity for describing collisionless high-β plasmas since the assumptions of isotropic pressure, negligible heat flow, and small ion Larmor radius are difficult to justify. It is therefore appropriate to apply an alternate and more soundly based approach to see to what extent MHD theory is applicable to high-β plasmas. The Maryland theory program in high-β plasmas emphasizes kinetic (Vlasov-Maxwell) studies of the equilibrium, stability, and anomalous transport properties of theta-pinch plasmas, and numerical studies of the implosion dynamics and sheath stability using fluid equations modified to include the anomalous transport produced by various cross-field instabilities. This paper summarizes several recent investigations in this program. These investigations include: (a) Studies of the Vlasov equilibrium and stability properties of linear theta-pinch systems. The j'th component equilibrium distribution function is of the form $f_j(H, P_\theta, v_z)$, where H, P_θ and v_z are the single-particle constants of the motion in the equilibrium fields. Equilibrium properties (such as density, temperature, and magnetic field profiles) are calculated for various choices of distribution function, with no assumption that the Larmor radius is small. Examples of sharp-boundary and diffuse-boundary equilibria are discussed for both parallel-biased and reverse-biased magnetic field. It is shown that $\partial f_j(H, P_\theta)/\partial H \le 0$ is a sufficient condition for stability to small-amplitude azimuthally symmetric ($\partial/\partial\theta = 0$) electrostatic perturbations. (b) Quasilinear studies of the anomalous thermal conductivity (parallel to the field lines) in finite-length turbulently-heated theta-pinch plasmas. Moment equations are obtained that describe the time evolution and axial profile of the thermal energy and heat loss. The competition between heat loss and turbulent heat production and the effect of turbulence in reducing the heat conductivity has been studied numerically for various cross-field instabilities. It is found that the contribution of turbulence to the thermal conductivity coefficient is much smaller than would be obtained if the anomalous resistivity (used to calculate heat production by instabilities) were also used to calculate thermal conductivity along field lines. (c) Numerical studies of theta-pinch implosion dynamics and sheath stability. The model[1] consists of a set of moment equations which include the effects of turbulence self-consistently through anomalous transport terms which evolve in space and time as various instabilities (ion acoustic, Buneman, and modified two-stream) develop and as the macroscopic parameters change. Results from numerical integration of these equations in cylindrical coordinates are in very good agreement with experimentally observed implosion dynamics. The code has also been used to investigate the stability properties of several equilibrium sheath configurations. For a sufficiently narrow sheath, the electron drift velocity is large enough to trigger various cross-field instabilities. In response to the turbulent field fluctuations it is found the characteristic sheath width increases to a (stable) value ΔL_s. The scaling of ΔL_s, with β, T_e/T_i, etc., is discussed.

†Research supported by Atomic Energy Commission Contract No. AT-(40-1)-4513.

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Numerical Determination of Mode Spectra in a Finite Straight Tokamak with Uniform Axial Current and Elliptic Cross Section*

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ABSTRACT

We have generalized a previously reported computational program[1] which gives the mode spectrum[2] for small perturbations from special low-pressure equilibria. The present generalization incorporates the effect of finite pressure and finite wave length but retains the constant axial current and similar elliptic magnetic surfaces of the previous calculation. This model is appropriate for studying ballooning effects associated with variations of field line curvature along a magnetic line. For this model we can determine the limits of the continua expected in the wave spectrum. We give a comparison of the numerical results with these predictions, as well as typical eigenmodes associated with instabilities.

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* Work supported by U. S. AEC Contract AT(11-1)-3073; use was made of computer facilities supported in part by NSF grant NSF-GP 579.

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TWO-DIMENSIONAL NUMERICAL SIMULATION OF HIGH-BETA PINCHES AND TOROIDS

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Two related but separate computational studies of high beta systems will be described: (1) the initial-phase motion of an axi-symmetric screw pinch or high-beta tokamak, and (2) the use of a magnetic pinching field (or dam) to contain the shock-heated plasma in a coaxial shock tube.

The latter problem has been solved in the r-z plane using a version of Marder's GAP code for the plasma, and an over-relaxation method for the vacuum field. Essentially, the plasma is governed by a set of hyperbolic equations, while the pinching or damming field is governed by a Laplace equation: the complete problem is a free-surface problem in which these two regions must be solved for jointly. Results of the simulation show that such a stuffing field scheme (originally proposed by Gross) is indeed a practical experiment, and realistic currents will produce useful pinching or damming effect.

The former problem is being studied as a two-dimensional problem in the poloidal plane, and axi-symmetry (i.e. toroidal direction) is assumed. The equations used are the dissipative single fluid MHD equations. The plasma is driven by external sources, given as known functions of time: first, the total current in the coils that produce the toroidal field, and second, the total current in the plasma that produce the poloidal and vertical fields. In tokamaks, the first current is essentially constant, while for screw-pinches, both are prescribed functions of time. The purpose of the study is to determine the time evolution of the cross-sectional shape and properties of the plasma, as determined by the geometry of the device and the given currents.

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2-D MHD Z-Pinch Calculations

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Abstract

Preliminary investigations of high density (>10^{18}/c.c.) Z-pinches have led to considering the time dependent development of sausage instabilities. Using a 2-D MHD code as a modeling tool, various configurations of plasma and magnetic field are studied. First, a sheet current system such as formed by "snow-plowing" is calculated showing the instabilities growing out of random noise and, after striking the axis, dispersing to a marginally stable state. Next, the actual formation of a pinch system from an overpressure magnetic field is examined and indicates instability formation only after the inward acceleration has ceased. Finally, a sheet current configuration imbedded in a dense gas is studied to determine the coupling of the instability with the gas. Observations of certain hydrodynamic and field quantities are made in an effort to shed light on past Z-pinch experiments and indicate directions for future work in the high density systems.

*Work performed under the auspices of the U.S. Atomic Energy Commission.

ANOMALOUS RESISTIVITY AND TRANSPORT DUE TO IMPURITIES IN HIGH TEMPERATURE PLASMAS*

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The problem of anomalous electrical resistivity and of transport due to impurities are considered for high temperature plasmas in which the electrons and/or the ions separate into trapped and circulating populations.

A current along the magnetic field is induced by a sufficiently small d.c. electric field that no appreciable deformation of the trapped electron distribution is produced by it. When this electric field is not too low in comparison with the critical runaway field the circulating electron distribution can be sufficiently displaced or distorted to induce the excitation of (odd) standing modes, along the magnetic field lines, which have zero electrostatic potential at the point of minimum magnetic field. Two groups of circulating electrons, having velocities of opposite sign, resonate with these modes and through them exchange momentum with the magnetic field. An anomalous resistivity is then produced for which the energy supplied by the electric field remains within the electron distribution while the momentum is absorbed by the magnetic field. These quantities are evaluated by developing a quasilinear theory that is appropriate for studying modes which affect simultaneously different portions of the electron distribution, and that takes the effects of magnetic shear into account. A similar model is developed for the purpose of inducing resistivity larger than classical (so-called catalytic) by exciting the same modes through a parametric coupling with modes induced by an external microwave source when the applied d.c. electric field is below the measuring field.

Anomalous thermal conductivity can be produced by odd standing modes which are excited in the presence of a relatively small concentration of impurity ions. In particular, these modes are found to be unstable, even when the impurity density gradient is in the same direction as that of the main ion population, such as in the case where impurities may be concentrated on the axis of the plasma column, if the relative temperature gradient of the main ion population is of the same order as a larger than the relative density gradient of the same population.

In regimes where the ion collision mean free path is shorter than the magnetic field connection length the impurity driven modes become fluid-like and depend on finite thermal conductivity, along the magnetic field, of the main ion population [1]. We expect that these collisional modes will manifest themselves with more macroscopic effects than the kinetic modes we have mentioned earlier.

* Supported in part by the U.S. Atomic Energy Commission.

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AN ANALYSIS OF TOKAMAK DISCHARGES MODELED WITH IMPURITIES*

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ABSTRACT

An analysis has been made of published data on ORMAK,[1] ST,[2], ATC,[3] T-3a[4] and T-4[5] tokamak discharges in hydrogen using various models of impurity constituents and distributions to eliminate the resistivity enhancement or resistance anomalies which are found if all the plasma ions are assumed to have $Z = 1$. The low-to-intermediate collision frequency transport theory of Hinton and Rosenbluth[6] was used to evaluate both the neoclassical conductivity reduction and the bootstrap current effects which modify the Spitzer resistivity. The calculations provide a one-dimensional snapshot view of a discharge in that they use a set of data self-consistent at a given time in the discharge, usually the time at which the electron temperature and density profiles are measured in a series of shots. The impurity ionization states are calculated under various assumptions as to the electron temperature and density evolution up to that time. In addition to the electron temperature and density profiles, the experimental data used in the calculations are: the plasma radius a, the major toroidal radius to the center of the discharge R_o, the limiter radius a_ℓ if it differs from a, the toroidal magnetic field B_ζ, the plasma current I_ζ, the plasma voltage V corrected for inductive effects, and the ion temperature profile or some model for it. Although much information has been published on tokamak discharges, in very few cases is the information complete enough to perform this type of calculation.

Summarizing the conclusions, with this discharge modeling we find Spitzer resistance anomalies ($R_{expt}/R_{Spitzer}$) ranging from ~ 2.3 to ~ 10 and neoclassical resistance anomalies (R_{expt}/R_{neo}) ranging from ~ 1.8 to ~ 6. The different impurity models yield results which differ considerably in detail and in some implications; however, there are some general similarities. First, if only low Z impurities such as carbon and/or oxygen are used to eliminate the neoclassical resistance anomalies, for Spitzer resistance anomalies $\geqslant 4$, the amounts of such impurities exceed 10% of the hydrogen density and lead to large central "proton deficiencies." Lesser amounts ($\leqslant 1\%$ of high Z impurities such as tungsten or gold (for ORMAK) accomplish the same results with much smaller "proton deficiencies." These amounts of impurities are found to be "collisional" ($\nu_{*Z} \geqslant 30$) over the whole of the discharges, which has significance in regard to the theoretical treatment of their transport. Finally, models in which the impurity concentrations are peaked near the center of the discharge can yield q (safety factor) profiles that are MHD unstable which appears to agree with recent observations on T-4.[5]

*Research sponsored by the U. S. Atomic Energy Commission under contract with Union Carbide Corporation

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NONLINEAR COUPLING OF KINK MODES AND THE DISRUPTIVE INSTABILITY OF THE TOKAMAK.

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- *Abstract* -

The $m = 2$, $n = 1$ kink mode is shown to be capable of destabilizing the $m = 1$, $n = 1$ internal kink. A nonlinear Lagrangian theory is developed for the coupling of modes of different pitch, and it is applied to the interaction of these modes. The coupling to the $m = 2$ mode provides the necessary boost to the internal mode and could make it account even quantitatively (where it had failed when considered by itself [1]) for many of the features of the disruptive instability.

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RECENT EXPERIMENTAL AND THEORETICAL WORK ON STRONG RELATIVSTIC ELECTRON RINGS (E-LAYERS) AT CORNELL* - H. A. Davis, H. H. Fleischmann, R. A. Meger, D. A. Phelps, A. C. Smith, Jr., D. M. Woodall (Laboratory of Plasma Studies, Cornell University, Ithaca, N. Y. 14850), H. L. Berk, and R. V. Lovelace (Plasma Physics Laboratory, Princeton University, Princeton, N. J. 08540).

Recent experimental and theoretical work in the RECE-program at Cornell has been concentrated in three areas: (A) The question of gross instabilities has been investigated by superimposing on the basic mirror field of RECE-Berta[1] a variety of perturbation fields which can be expected to enhance incipient large-scale instabilities. No indication of such instabilities was observed for perturbation fields of up to about 1% of the mirror field. The case of a magnetic field gradient across the diameter of the electron ring was investigated in detail. The radial shift of the electron rings observed experimentally shows good agreement with the radial shift of the equilibrium position theoretically predicted using newly calculated values of the image field indices. Further favorable evidence for gross stability of these electron rings was obtained from two other experiments in which lifetimes of up to 180 μsec (in the larger RECE-Christa device) and 80 μsec (in a small cusp injection experiment[2]) were observed (corresponding to about 10^5 electron orbits). (B) To determine the effects of possible microinstabilities, the time dependence of the average energy of the fast electrons was measured by X-ray absorption spectroscopy. The observed energy loss rates are consistent with collisional losses of the electrons in the gas, which is in accord with the observed collisional decay of the layers. An order of magnitude analysis of the existing conditions suggests the possiblity of a slight linear two-stream instability. (C) A conceptual study was performed on the generation of high-energy (100 - 300 MeV) ion rings (protons or deuterons) by low-current (about 10 A) injection of 10-20 MeV H_2^+ ions into a preformed electron ring (possibly avoided by the addition of quadrupole fields) and subsequent compression. If feasible, a scheme of this type could lead to a number of important advantages in a fusion reactor.

* Work supported by the Atomic Energy Commission under contract No. AT(11-1)-2319 and by the Empire State Electric Energy Research Corporation.

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Plasma Diffusion for a Two-Dimensional Model

by

John M. Dawson

Recently there has been a good deal of interest in diffusion exhibited by two-dimensional plasma models. Much of this interest stems from the fact that these models exhibit anomalous transport. Theories have been developed which explain the anomalous transport in terms of thermally excited convective turbulence. According to this theory the decay time for an eddy of wavenumber k is $(k^2D)^{-1}$ where D is the turbulent particle diffusion coefficient. Recent numerical simulations of Okuda show that the lifetime of the eddies $(k^2D^{-1}(W_T/W_P$, where W_T is the total energy of the eddy and W_P is the fraction carried as kinetic energy of the particles. This becomes particularly long at high magnetic fields and is much longer than the time required for particles making up an eddy to diffuse away and be replaced by new ones. A discussion of how this can occur physically will be given along with some numerical calculations which illustrate the process in the guiding center limit. One result of this study is that for an observer moving with a particle short wavelength fluctuations do not die out in times of the order $(k^2D)^{-1}$ but rather have a lifetime which is independent of k. The consequences of this for the Taylor McNamara theory of diffusion in a two-dimensional guiding center plasma will be given.

Radiofrequency Confinement of an Over-Dense Plasma Produced by Laser-Pellet Interaction*

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Radiofrequency confinement of a high density plasma has never been demonstrated. However, recent accomplishments both in experimental verification[1] and numerical simulation[2] of confinement of low density plasmas by means of rf fields points to the possibility of the realization of confinement of plasmas whose densities are sufficiently high so as to exclude the rf field.

RF confinement of low density plasmas is based upon the oscillation center approximation in which an rf field, usually set up in an appropriate microwave cavity,[3] exhibits a quasi-potential confinement well, such as the TE_{111} mode of a cubical cavity. The quasipotential ϕ is defined by the expression

$$\phi = \frac{q^2 E_o^2}{4m\omega^2}$$

and is inversely proportional to the mass of the particles and directly proportional to the square of the electric field. Numerical simulation of this model by means of particle-in-cell codes showed good evidence of steady state confinement.

For the confinement of high density plasmas, it is required that the energy density of the external rf fields in the cavity outside the plasma be greater than the kinetic energy density of the plasma itself.[4] For thermonuclear reactors, confinement of high temperature, high density plasmas requires rf electric field intensities or approximately 10^6 volts/cm.

This strong field intensity is very difficult to achieve in non-superconducting cavities, because of the tremendous power that is dissipated in the walls of the cavity. However, lower field intensities can be used to confine high density plasmas of temperatures slightly below that required for thermonuclear fusion. When previous experiments were attempted however, the confining mode structure was destroyed because of breakdown of the gas filling the cavity in regions of high field strength outside the confinement potential well.

A method of resolving this difficulty is to utilize a laser-produced plasma placed in the center of the potential well. A 2-10 joule ruby laser may be Q-switched to allow vaporization of a small lithium deuteride pellet. The rf fields can then produce ionization and confinement of the expanding vapor. Since the cavity is at low base pressure, there will be no breakdown outside the potential well region and verification of the confinement theory should be possible to observe.

*Work supported by the U.S. Atomic Energy Commission and the National Science Foundation.

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Interchange Instability in Non-Circular Cross-Section Models

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Axially symmetric toroidal MHD equilibria with arbitrary non-circular cross-section, where the current density is independent of the poloidal flux, are examined for stability against the localized interchange by analytical and numerical means. The effects of magnetic well, shear, magnetic surface shape and finite β are exhibited and compared for various cross-sections.

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STABILIZATION OF THE TRAPPED ION DISSIPATIVE INSTABILITY BY IMPURITIES

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ABSTRACT

The influence of impurities on the trapped ion dissipative instability is extensively investigated. It is found that impurities lead to most significant stabilizing effects on the dissipative modes on trapped ions. In particular, when the impurity ions are in the banana regime, sufficient conditions for complete stabilization of the instability have been found in the case of sharp impurity density gradients. When these conditions are not satisfied, one finds nevertheless a noticeable shrinking of the range of unstable wave numbers upon increasing impurity concentration. These features have been discussed quantitatively, in terms of the impurity parameters, both with reference to present day experiments and for future experiments with reduced impurity levels.

THEORETICAL PROBLEMS IN ELECTRON BEAM FUSION*

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Pellet heating by intense relativistic electron beams has been proposed[1,2] as an alternative to the more familiar laser powered pulsed fusion schemes. Our interest in electron beam fusion is supported by a number of important experimental results. The previously reported focusing experiments[2] at the 100-1000 J level have recently been successfully scaled to the 10 kJ level.[3] These experiments were performed on the 1 MV Hydra accelerator using a plasma injection[4] focusing technique. Another focusing method employing a sharp-edged emitter has been used to focus the Hydra beam onto small hemishells. Initial results indicate that most of the available energy was deposited on the target with no obvious gross asymmetry.

Theoretical problems in e-beam fusion can be categorized under focusing, diode plasma production and behavior, beam deposition and interaction, and target configuration and dynamics. The initial focusing calculations have been extended to diodes with large openings in the cathode to facilitate cathode survival. The computational results, supported by experimental data, indicate that "hollow" diodes focus quite similarly to the standard solid cathode diodes.

Extensive calculations employing a 2-D materials response code have been performed in an attempt to correlate anode damage patterns with the e-beam input parameters. Variations of the assumed focal diameter, pulse length, machine voltage, and total deposited energy have indicated that the crater damage correlates strongly with the deposited energy. These calculations have provided a useful diagnostic for focused beam experiments. Results for both the plain anode and for the hemishell irradiation experiments will be shown.

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Resistive Instabilities in General Toroidal Configurations*

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ABSTRACT

We extend an earlier calculation[1] of localized resistive interchanges to include finite pressure effects. We find two additional destabilizing terms in the stability criterion for these modes, which vanish for highly symmetric configurations like circular cylinders as well as in the low pressure limit. Our treatment also leads to a new derivation of the Mercier criterion for ideal interchanges and clarifies the relationship between the ideal and resistive criteria. Finally, a systematic study of the dispersion relation, using Nyquist techniques to allow for complex frequencies, indicates new properties of the tearing mode. This mode is either stabilized or converted from a purely growing to a primarily oscillatory instability by sufficiently favorable average curvature. Furthermore, when the growth rate is real, it depends on the resistivity as $\eta^{(3-2H)/(5+2H)}$, where

$$H \equiv [\langle \sigma B^2/|\nabla V|^2\rangle - \langle\sigma B^2\rangle\langle B^2/|\nabla V|^2\rangle/\langle B^2\rangle] / \Lambda ,$$

with $\sigma = \vec{J}\cdot\vec{B}/B^2$, Λ the shear parameter, and brackets denoting the field line average of Ref. 1. In previous work, $H = 0$.

* Supported by U. S. Atomic Energy Commission Contract AT(11-1)-3073.

† On loan from Westinghouse Research Laboratories.

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Spin, Parity, and the Spectrum of Ideal Magnetohydrodynamics

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Abstract

The presence of three waves in homogeneous ideal magnetohydrodynamic systems and the splitting of these three waves in five discrete modes in inhomogeneous systems is explained in terms of the spin and parity which can be associated with magnetohydrodynamic disturbances. As in quantum electrodynamics of spin one particles the basic equation of ideal MHD is a sixth order system of partial differential equations for a vector field $\underset{\sim}{\xi}$. In the same formal manner one can therefore introduce $S = 1$ spin operators acting on the components of $\underset{\sim}{\xi}$. This description properly leads to the three ideal MHD waves. More important, the magnetic properties of these waves are what is to be expected from spin-one waves, viz. antiparallel orientation of the perturbed magnetic field with respect to the unperturbed field for the slow wave, perpendicular for the Alfvén wave, and parallel for the fast wave.

In inhomogeneous systems, like the diffuse linear pinch, the reflection symmetry of homogeneous systems is broken. This leads to a splitting of the slow and the Alfvén waves in right-handed and left handed waves having different frequencies, whereas the fast waves remain degenerate. The thus obtained five modes constitute five discrete subspectra which accumulate from below (Sturmian) or above (anti-Sturmian) at the cusp and Alfvén-continua, whereas the fast wave subspectrum accumulates at $\omega^2 = \infty$.

It appears that these global properties of ideal MHD will be useful for the description of waves and instabilities in complicated geometries, where the use of perturbation theory demands an explicit knowledge of a simpler unperturbed system.

MHD Equilibrium for a Bumpy Axisymmetric Plasma Column with a Free Boundary*

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Solutions to the nonlinear ideal MHD equilibrium equations are computed for the case of an infinitely long axisymmetric cylindrical plasma column. Periodic boundary conditions along the axis of the column are used to approximate a torus in the limit of infinite aspect ratio. The applied axial magnetic field is made to have sinusoidal variations, giving rise to "bumpy" flux surfaces. No restrictions are made on the amplitude or wavelength of these variations. The problem is bounded by a perfectly conducting wall of constant radius r_w. A circular limiter of radius $r_L \leq r_w$ at a fixed axial position is utilized to permit a free plasma-vacuum boundary. The limiter produces a vacuum region between the plasma and the wall with the plasma-vacuum interface being the surface of constant flux corresponding to the value of the flux at the limiter. The location of this interface must be found as part of the problem.[1] Currents are allowed in both the axial direction and in the direction of the ignorable coordinate. Solutions to the differential equations for ψ are computed using the successive relaxation technique.[1,2] The bumpiness in the flux surfaces will decrease to zero at $r = 0$. The rate of decrease of the bumpiness with decreasing r is observed to be much greater with the conducting wall than in calculations without a conducting wall present.[3] It is also observed that increases in plasma β and axial current allow the bumpiness to penetrate deeper into the plasma. The calculations are applied to several existing and proposed tokamak experiments and show that bumpiness penetrates less as the ratio of the plasma radius to the spacing between the magnet coils increases.

*Work supported by the National Science Foundation and the Atomic Energy Commission.

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The Fat Mirror Machine

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Abstract

It has been shown that islands can be induced by self-consistent plasma currents. It is also possible to develop pressure islands even though the magnetic field structure remains simple. The basic mathematical cause is a continuous spectrum which extends to $\omega^2 = 0$. Pressure islands in general closed line systems have been studied for perturbations with $m > 0$ (in particular for a toroidal perturbation, $m = 1$). We examine, here, islands induced by an azimuthally symmetric perturbation ($m = 0$) in a simple, axially symmetric mirror machine,

$$\Delta^*\psi = -r^2 p'(\psi).$$

An essentially complete solution can be obtained by considering slow, "adiabatic", variation in z; locally $\Delta^*\psi = r(\psi_r/r)_r$. The function $p(\psi)$ is fixed as the outer boundary is varied, $r = a(\varepsilon z)$. For a given profile $p(\psi)$, there is a maximum ratio a_2/a_1 of plasma radii, beyond which a reversed field (island) must occur on axis. Equilibria with islands have, of course, been studied before. The present islands are distinctive because they are not imposed; the plasma decides that they must appear whenever the plasma is sufficiently wide. These islated equilibria are essentially high β.

The phenomenon of islation is entirely distinct from bifurcation, which occurs at a different critical radius. The island equilibrum is unique, given $p(\psi)$; bifurcation implies the presence of several solutions with the same $p(\psi)$. A qualitative examination of these high β equilibria indicates the possibility of better MHD stability than simple mirrors (although one special case, with p = const. within the island, has been shown to be unstable - private communication, W. Newcomb).

Adiabatic Evolution of Islands

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Abstract

Islands occur by design (Doublet) or by accident (Tokamak). They **evolve** in time in response to variation of external currents or by **intrinsic** plasma processes. In many cases the time variation is slow **compared** to Alfven wave equalization and fast compared to any dissipative processes. We consider the time evolution of perfectly conducting MHD equilibria subject to adiabatic constraints, including simple **geometries** as well as formation, growth, merging, and disappearance of **islands**.

In two dimensions,

(1) $$\Delta\psi = - p'(\psi),$$

where $p(\psi)$ is not given but is determined by $pq^\gamma = (\psi)$, where $\mu(\psi)$ is given ($q=\oint d\ell/B$). With μ given instead of p, the usual iteration of (1) cannot be expected to converge. One difficulty is that the elliptic equation (1) allows specification of only one value of ψ on the boundary, whereas the adiabatic constraint implies prior specification of ψ at the center of each island (this is a difficulty in even the simplest problem, with a single interior minimum).

We describe analytic and numerical techniques which are appropriate for the solution of such problems. A crucial point is the formulation of conservation laws (jump conditions) connecting adjoining regions. The situation is analogous to the classical shock problem. In particular, we can calculate mixing, changes of entropy, etc. entirely within a perfectly conducting MHD framework. The time scale for these irreversible (line-breaking, mixing, etc.) processes has no relation to the value of the resistivity or any other dissipation coefficient; it is governed entirely by the external rate of variation of constraints. In some cases (primarily growing islands) the configuration is path independent; in other cases (primarily shrinking islands), the exact time evolution of the constraints must be followed.

This analysis applies equally to varying static equilibria and to traveling waves. The theory is a prerequisite to any rational treatment of transient diffusion or heat transfer in a geometry more complicated than a straight, one dimensional, circular cylinder.

The Bumpy Tokamak

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Abstract

A low β, closed line equilibrium is characterized by the two properties,

(1a) $$J \sim p \sim \varepsilon$$

(1b) $$p = p(q_0) , \quad q_0 = \int d\ell/B_0 .$$

An axially symmetric, low β Tokamak,

(2) $$\Delta^*\psi = - r^2p' - ff' , \quad f = rB_\theta$$

is characterized by the scheme

(3a) $$B = B_0 + \varepsilon B_1 + \varepsilon^2 B_2$$

(3b) $$J = \varepsilon J_1 + \varepsilon^2 J_2$$

(3c) $$p = \varepsilon^2 p$$

B_0, B_1, B_2 are alternately toroidal and poloidal and $p \neq p(q_0)$. We call a low β , low shear equilibrium <u>natural</u> if $p = p(q_0)$, unnatural otherwise. By generalizing the Tokamak equilibrium to be a perturbation of an arbitrary closed **line**, vacuum field, $B_0 = \nabla\alpha \times \nabla\beta$, with reflection symmetry, instead of the toroidal $B_0 \sim 1/r$, we obtain a continuous transition between natural equilibria which require $J \sim \beta$ to support the plasma, to the stronger current, $J \sim \sqrt{\beta}$, required to support general p-contours, not related to q_0. An unnatural generalized Tokamak is characterized by the same scheme (3), where B_0, B_1, B_2 have alternately even and odd partiy instead of toroidal, poloidal. Replacing (2), we have

$$\sigma(\alpha,\beta) = - q_0p'(\psi_0) - F(\psi_0)$$

where $J_1 = \sigma B_0$.

Turning a closed line vacuum field with closed q_0 contours (e.g. Bumpy Torus) into a Tokamak offers the possibility of higher β (or lower toroidal current) than a conventional Tokamak.

High Beta Toroidal Belt Pinch Stability

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Abstract

The original intent behind the belt pinch experiment (originated at Garching in 1970 by R. Wilhelm) was an attempt to produce a stable, high-β ($\beta \sim 50\%$), toroidal plasma with total toroidal plasma currents which in the case of the usual circular cross sections would yield instabilities. The guiding argument supporting these efforts was that by holding the toroidal magnetic field, toroidal current and aspect ratio (ratio of torus radius to plasma minor radius) constant, the value of "q", the safety factor, increases as H^2/a, where H is the effective plasma height and a is the plasma minor radius. It was recently shown by Freidberg and Haas[1] for $\beta \sim \varepsilon < 1$ (ε is the inverse aspect ratio) that above a certain value of β the plasma is unstable for any value of q. This occurs both for circular and non-circular plasma cross-sections. Using the theory of F and H for $\beta \sim O(1)$ plasma and using current belt pinch plasma parameters numerical calculations of kink mode stability show that the belt-pinch is limited to β values on the order of a few percent. The role of q for the unstable modes in some sense dictates the mode structure. For example, for a β of 2.5 % and plasma major to minor radius of 5, a q value less than 1 leads to a predominantly m=1 like mode. Stability for this case is reached only at very large q.

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1) Freidberg, J.P., Haas, F.A., "Kink Instabilities in a High-β Tokamak with Elliptic Cross Section, to be published in Phys. Fluid."

OPTIMAL INJECTION OF BEAMS IN A MIRROR WELL*

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ABSTRACT

An anisotropic finite-β plasma trapped in a mirror well is subject to local MHD instabilities which depend strongly on the shape of its angular distribution function. Moreover, the angular distribution, itself a solution of a diffusion equation, can be significantly modified by appropriate changes in the relative positions and amplitudes of injected beams used to create or maintain the plasma. Thus the question is posed: how optimally to inject so as to maintain the greatest overall density, without becoming unstable to local MHD modes, and without inordinate demands upon the technology of beam injection.[1]

The essence of the problem is such that it is not susceptible to optimization by ordinary procedures of functional differentiation. Instead, it can be solved as a problem in the theory of continuous games (with Nature, or the budget director, as our opponent). One can then use the methods of linear programming in conjunction with computer solutions of the Fokker-Planck equation to determine the optimal strategy in the general case.

For many purposes, an adequate modelling of the process of angular diffusion is given by the following equation for the angular distribution function $g(\mu)$,

$$\nu_c (d/d\mu)[(1-\mu^2)dg/d\mu] + S(\mu) = 0 ,$$

where $S(\mu)$ is the injected current at pitch angle $\theta = \cos^{-1}\mu_0$. Here ν_c is an appropriate collision frequency and $\mu^2 < \mu_0^2$ where $\theta_0 = \cos^{-1}\mu_0$ is the loss-cone angle, $g(\pm\mu_0) = 0$. No essential complication is introduced by the addition of a loss term $-\lambda(\mu)$ [e.g., $-\lambda g(\mu)$], modelling the loss of ions due to electron cooling. With these simplifications, the problem can be solved analytically under a variety of restrictions as to the nature of the available beams, the essential feature being that the instability criterion is marginally satisfied over the widest possible angular range. A figure of merit is introduced which is appropriate to general experiment or reactor design, and the optimal strategies for this and a number of other games are determined and described.

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*Work performed under the auspices of the U. S. Atomic Energy Agency

[1] The suggestion that tailoring the injection can increase the stability threshold is due to R. F. Post.

The Topolotron: A New Concept in Magnetic Confinement

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Abstract

A new design for a toroidal magnetic confinement device is proposed, based on an idea by Robert W. Bass. We assume unity beta, axial symmetry, and topological stability of the magnetic field line portrait on the surface of the plasma. Topological stability is required if the field line portrait is to be insensitive to small variations in the system. These conditions lead to a unique device, called the Topolotron. Although topological stability is basically different from dynamical stability, recent calculations indicate that the Topolotron may have excellent dynamical stability characteristics. It is expected to be stable against magnetic braiding. The solution of the differential equation of the system exists in a neighborhood of the plasma. Numerical work, showing the field lines, and some features of the design will be presented

Conceptual Design of a Controlled Thermonuclear Reactor with the Adiabatic RF Plugging Scheme

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We consider a linear machine with cusp geometries on both ends. At the line and point cusps, the adiabatic rf plugging is applied. Cusp configuration is convenient for the rf plugging, because in the line (point) cusp region the width (radius) of the plasma can be reduced down to the order of the ion Larmor radius.[1] There exists an eigen mode[2] near the ion cyclotron frequency in such a thin (slender) plasma, so that an external field can penetrate inside the plasma.

Six conditions for the adiabatic motion are given, which lead to a concept of the critical energy[3], the height of the adiabatic barrier proportional to $E_0^{8/5}$, E_0 being the strength of the rf field in the plasma. A numerical computation of confinement time under the rf barrier guarantees that the particle confinement becomes more than 10^4 times longer than the transit time. This implies that if the linear dimension of the scheme is assumed to be about a hundred meters the required strength E_0 is quite within the limit of the present technology.

This scheme does not suffer from both the end loss and the micro instability. The length and the density of the plasma of this scheme are much less than those of the multiple mirror. It is also noted that the rf barrier plays a role of an effective diverter, since impurity ions and high energy H_e^{+2} easily go over the barrier.

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1) T, Hatori: Institute of Plasma Physics, Nagoya, Rep. No. IPPJ-171, 1973.

2) T. Watanabe and T. Hatori: IPPJ-132, 1972; T. Hatori and T. Watanabe: IPPJ-133, 1972.

3) T. Hatori and T. Watanabe: IPPJ-182, 1974.

KINETIC THEORY OF PLASMA SCRAPE-OFF IN A DIVERTOR TOKAMAK

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Presuming an axisymmetric divertor configuration (poloidal divertor), we study the spatial decay of plasma density, and the electrostatic potential variation, in the vicinity of the divertor separatrix, using kinetic theory. We suppose $T_i >> T_e$, as suggested by transport considerations, and treat a model in which the electrons are collisional, the ions collisionless. Thus n_e is determined from the ion drift surfaces, while $n_e \propto \exp(e\Phi/T_e)$. The resulting scrape-off layer equilibrium is characterized by moderate electric fields, $cEr/B << (2T_i/M_i)^{1/2}$, and a thickness of the order of a few ion poloidal gyro-radii. Explicit, approximate, expressions for $n(r,\theta)$ and $\Phi(r,\theta)$ are derived, in terms of the density, n_o, at the inside of the scrape-off layer.

Particle and energy loss to the divertor collector plates are determined by collisional scattering of ions onto diverted orbits, analagously to scattering into the loss cone of a mirror machine. The loss rates are evaluated in terms of a phenomenological angular momentum diffusion coefficient, by solution of the appropriate Wiener-Hopf problem. A matching condition applied to the Wiener-Hopf problem then provides an estimate of n_o, and also the boundary conditions required for solution of the radial transport equations in the plasma interior. Finally we compute the sheath potential at the collector plate, from the usual requirement of ambipolar loss.

VLASOV TREATMENT OF BALLOONING MODES IN HOT ELECTRON PLASMAS*

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ABSTRACT

Stability criteria, derived from δW for both ideal MHD and guiding-center plasma models, have been evaluated for the finite beta equilibria expected in the ELMO Bumpy Torus (EBT). These criteria indicate that ballooning modes can be unstable due to unfavorable curvature of the magnetic field lines. For hot electron plasmas, such as EBT, the drift frequencies are typically greater than ion gyrofrequencies, and the applicability of these fluid models, as well as previous treatments using the Maxwell and Vlasov equations, is questionable. Thus we have used a Maxwell-Vlasov treatment for a slab model in which $\vec{E}$ is perpendicular to $\vec{B}_0$ and the frequency can exceed the ion gyrofrequency. In the appropriate limit this treatment gives the proper fluid equations of motion. (The driving forces for this mode are those associated with guiding center drift motion and $\vec{j}_0 \times \delta\vec{B}_{\parallel}$.) Using the more general dispersion relation we have deduced various stability boundaries. For EBT the most significant result is that the mode is stabilized when the guiding-center drift frequency exceeds the ion gyrofrequency by a factor of order unity depending on ω_{pi}/ck. In this analysis we have neglected dispersion in the electron drift speed and treated the ions as cold. Relaxing these approximations seems unlikely to give rise to any rapidly growing waves, although some microscopic instabilities may be possible. This question is under continuing study.

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PARAMETRIC SURVEY FOR THE LINEAR TWO-COMPONENT SYSTEM*

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ABSTRACT

A parametric study is presented for the linear two-component mirror fusion system[1] in which the interelationship of the important system parameters is displayed. The distribution function of the high energy component and related functions are calculated from the Fokker-Planck equation. These functions are conveniently parametrized by the ratio of injection energy to the target electron temperature. The Q values for the two component system are found to be increasing functions of the injection energy up to 200 keV and then to exhibit a broad maximum from 200 keV to 600 keV. The optimum Q values are significantly greater (>2) than for the conventional mirror system. System lengths are derived as function of the magnetic field, target electron temperature, mirror ratio, and α particle heating. For a given Q (or injection energy/electron temperature ratio) the system lengths, which may be long, are balanced against magnetic field values, which may be large. Effective bounds on several parameters are established. Some problems are posed, including the attenuation of the incident beam on the deuterium "halo" surrounding the target plasma.

*Work performed under the auspices of the U.S. Atomic Energy Commission

[1]R. F. Post, T. K. Fowler, J. Killeen, and A. A. Mirin, Phys. Rev. Lett. 31, 280 (1973)

RECENT ADVANCES IN ION BEAM PROBING*

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Ion beam probing is an attractive diagnostic technique that provides space and time resolved measurements of several important parameters in energetic magnetoplasmas. The technique involves directing a beam of singly charged ions through the plasma and measuring the intensity and energy of doubly charged ions created by electron impact. All measurements are made continuously in time, and spatial resolution is provided by the different trajectories followed by singly and doubly charged ions in the confining magnetic field. In this paper we will present several new refinements in ion beam probing that extend the usefulness and validity of this diagnostic technique. In addition we will discuss our preliminary evaluation of beam probe systems for future CTR devices, and present the results of a detailed design study for a system which will be installed on the "LITE" device at UARL.

In the past year we have added electron temperature to the list of parameters measurable with an ion beam probe (n_e, ϕ, J). We have developed a multiple beam technique for sequentially probing the same volume of the plasma with ion beams of different species. The ratio of the normalized currents of doubly charged ions allows a direct evaluation of T_e if the ionization cross sections are known. Quantitative measurements of T_e up to nearly 1 KeV can be made using a Na^+/K^+ system. There is also a possibility of measuring T_i in fusion-related plasmas by taking advantage of ion-ion charge exchange reactions. At the present time this is pure speculation since there is essentially no information on ion-ion charge exchange cross-sections.

We have also made three important refinements in the measurement of plasma space potential. A DC offset in the feedback controlled electrostatic energy analyzer has been eliminated using the multiple beam technique. The voltage correction function arising from a change in the angle at which the beam enters the analyzer has been experimentally verified. Finally, the frequency response of the complete analyzer system has been extended to 100 KHz. With these refinements we can accurately calculate the electric fields inside a plasma, and also investigate coherent instabilities. In particular we have been able to measure the change in phase between density and potential oscillations as a function of position in the plasma.

We have shown that beam probe diagnostic systems can be used in the next generation of CTR devices once expertise in handling MeV ion beams is obtained. A system for a full-scale fusion reactor will require trade-offs between beam energy and spatial resolution in order to be practical.

The "baseball" magnetic field of the "LITE" device is the most complicated field ever analyzed for beam probe diagnostics. We have successfully developed detailed grid maps which will allow investigation of the minor cross section of the mirror-trapped plasma, and the steady state plasma after neutral injection. We are presently building a complete diagnostic system for installation on the "LITE' device.

*Work supported in part by AEC under Contract AT(11-1)-2229.

ABSTRACT SUBMITTED TO ANNUAL MEETING ON THEORETICAL ASPECTS OF CONTROLLED THERMONUCLEAR RESEARCH

THEORY AND EXPERIMENT ON LASER HEATED PINCHES AND SOLENOIDS[+]

A. L. Hoffman,[*] G. C. Vlases,[**] H. Rutkowski,[**] H. Hoida[**]

Classical plasma inverse-Bremsstrahlung absorption of intense laser radiation produces strong heating which leads to the formation and maintenance of beam trapping density profiles, permitting small diameter laser beam propagation over long distances. A longitudinal, two temperature model shows the propagation of heating waves into plasma columns, while radial calculations display on-axis density minimums and radial loss rates. Both of these calculated effects have been observed experimentally in theta-pinches and solenoids.

In 10-cm long, 2-cm bore theta pinch experiments, favorable density profiles were created by implosion and trapping of initial bias fields. A 40 kG magnetic field rose to peak strength in 500 nsec and, with initial fill pressures of 1 torr H_2, created a relatively low β, 10 ev plasma at an electron density of $6 \times 10^{17}/cm^3$. The 6-mm diameter plasma column was irradiated with up to 50 J of 10.6-μ laser radiation in a 70 nsec spike, and an on-axis electron density minimum ($3 \times 10^{17}/cm^3$) was measured by longitudinal interferometry. The plasma expansion reduced the internal magnetic field, and temperatures of 50 ev were inferred from the increased β and measured density. The heating wave propagation was observed by monitoring impurity line radiation from highly stripped ions, and beam trapping was revealed by laser burn patterns beyond the plasma column when the laser energy was high enough for the heating wave to completely traverse the plasma column.

Beam trapping and wave propagation were also observed in a laser irradiated solenoid at fields up to 150 kG, and filling pressures of 5 to 20 torr. Laser breakdown waves were studied holographically and with side-on image converter photography. These waves propagate several centimeters in both directions from the focal spot with laser pulse energies of 50 to 100 joules.

[+]Supported by AEC and DNA.

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Radial Normal Mode Stability Theory of Trapped Electron Mode

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For frequencies between the ion and electron bounce frequencies the drift wave occurs as a normal mode localized to the rational surfaces $m=\ell q(r_i)$ with widths of order $\Delta r \simeq \rho/S^{\frac{1}{2}}$ where $\rho = c(M_i T_e/eB)^{\frac{1}{2}}$, and $S = (\epsilon/q)(d\ln q/d\ln n)$. The destabilization occurs through the mechanisms of trapped electron collisions and bounce and drift resonances, where the dominant mechanism varies with $k_\perp \rho$ and ν_e^*. The normal mode equation is obtained from the Vlasov equation assuming for the ions that $\mathrm{Im}(\omega) > \omega_{bi}/\pi$ and that $k_x \rho_i < 1$. The electron distribution function is calculated for general ω/ω_{be} by introducing an approximation for the bounce orbits and for $\nu_e^* < 1$ by observing that the trapped electron collisions dominate and the collision model of Kadomtsev and Pogutse is adequate. We show explicitly the transition from $\omega/\omega_{be} > 1$ where

$$\mathrm{Im}(\tilde{n}_e^{tr}) \propto (\pi/2)^{\frac{1}{2}} [\omega - \omega_e^* (1 - \eta_e/2)] / |k_\parallel| v_e$$

to $\omega/\omega_{be} < 1$ where

$$\tilde{n}_e^{tr} / n_o = \epsilon^{\frac{1}{2}} \Sigma Y_n (e\tilde{\varphi}/T_e)$$

with

$$Y_n = \langle [(\omega - \omega_T^*) / (\omega - n\omega_b + i\nu)] J_n^2 [|m - \ell q(r)| \theta_o (v_\parallel/v)] \rangle$$

which is computed numerically in the transitional regimes. The modes with $(r_n/L_c)(T_i/T_e)^{\frac{1}{2}} < k_\perp \rho < (r_n/L_c)(\epsilon m_i/m_e)^{\frac{1}{2}} \nu_e^*$ are collision dominated, and we obtain

$$\gamma_k = \frac{\omega_o^2}{\omega_* \Gamma_o} \left\{ \epsilon^{\frac{1}{2}} \mathrm{Im}\, Y_o - S\left(\frac{\omega_*}{\omega_o}\right) \left| \frac{d \ln \Gamma_o}{d b} \right| (1 + T_i/T_e) \cdot \left[1 + \frac{\epsilon}{2} \left(\frac{L_c \omega_o}{c_s}\right)^2 \left| \frac{d\Gamma_o}{d b} \right| \frac{|R_e Y_o|}{(1 + T_i/T_e)} \right]^{\frac{1}{2}} \right\},$$

where $\Gamma_o(b) = e^{-b} I_o(b)$, $b = k_\perp^2 \rho_i^2$. The limit $\nu_e > \omega_*$ where $\mathrm{Im} Y_o \propto (\eta_e \omega_*/\nu_e)\langle J_o^2 \rangle$ is compared to the Fokker-Planck result and shown to be in reasonable agreement. For the modes with $\omega_* > \nu_e$, the real part of Y_o is shown to steepen the radial anti-well and increase the effectiveness of shear stabilization. For $\eta_e > 0$ ion inertia competes with the stabilizing η_e, but skin-effect profiles ($\eta_e < 0$) are strongly unstable. For short wavelengths, $k_\perp \rho \sim 1$, and low collisionality, $\nu_e^* \ll 1$, the normal mode has an average $k_\parallel$ such that $\bar{k}_\parallel L_c > 1$ in which case contributions arise from both $\mathrm{Im} Y_o$ and $\mathrm{Im} Y_1$-- the collisionally broadened electron Landau bounce resonance. The critical shear formulae for these modes will be given explicitly. Using the analytic solutions described above as guides, we study the important cases further with numerical integrations of the radial mode equations.

Classical Diffusion and Equilibrium in a Cylinder

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Abstract

Transport phenomena in a Tokamak are often studied with the assumption of quasi-static field (Curl E = 0). This is justified with present Tokamak parameters ($\beta < \alpha = (m_-/m_+)^{1/2}$) after the skin effect has decayed. Careful study of a straight cylinder based on Braginnskii's equations shows that there are four time scales corresponding to diffusion coefficients $D_0 = \eta/\mu_0$, $D_1 = \beta D_0/\alpha$, $D_2 = \beta^2 D_0/\alpha$ and $D_3 = \beta D_0$. With future Tokamak parameters ($\beta > \alpha$), the skin time D_0 is overtaken by the intermediate time D_1 or even by D_2, while the plasma may start to diffuse before the classical diffusion time D_3 sets in. Use of Kolodner's equations with more complete coefficients than Braginnskii's gives approximately the same results. This is surprising since there are very important differences between the two systems in the shock structure problem.

Equilibrium solutions with Ohmic heating and thermal conduction are also studied. For $\beta < \alpha$, the pressure profile is governed by the well known Lane-Emden equation for stellar structure in cylindrical geometry, and the plasma is contained within a finite radius when the temperature ratio $T_-/T_+ > 2$. For $T_-/T_+ < 2$, there is no strict equilibrium, but similarity solutions for the growth and decay in time can be found.

Two-Component Fusion Reactors with Sustained Suprathermal-Ion Energy*

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ABSTRACT

The energy multiplication factor of the D-T two-energy-component reactor scheme[1] can be enhanced by a factor of about two if the suprathermal ion energy is "clamped" at the value W_o that gives maximal ratio of fusion reaction rate to plasma drag. The optimal energy W_o is in the range 105 to 145 keV (depending on T_e), significantly lower than the optimal energy in the conventional scheme.[1] To realize the maximum gain, the clamping period must be many times the "slowing-down time," or else the suprathermal energy must be recovered (e.g., by magnetic decompression at the end of the clamping period).

In a tokamak device there are several possible techniques for accomplishing energy clamping: (1) Slow major radius compression. (2) Use of the toroidal electric field to maintain the energy of ions injected parallel to it. (3) Rigid displacement of the plasma column at a frequency related to the ion transit frequency around the torus. (4) RF heating at a harmonic of the ion-cyclotron frequency. (5) Energy input from driven or spontaneous Alfven waves.

On the basis of successful results from the ATC device, the first method seems quite feasible, and could lead, for example, to a "break-even" device with $T_e = 3.0$ keV, compression ratio 2.3, and $n\tau \sim 5 \times 10^{12} cm^{-3}$ sec.

*Work supported by U. S. Atomic Energy Commission Contract AT(11-1)-3073.

[1]J. M. Dawson, H. P. Furth, F. H. Tenney, Phys. Rev. Letters 26, 1156 (1971).

Equilibrium Theory - Toroidal Belt in a Force Free Field

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Abstract

Earlier computations of a plasma column equilibrium supported by a force-free magnetic field have not considered the possibility of the inaccessibility of the equilibrium (except, possibly, for stability calculations of the configuration). Applying topological constraints on the motion of finite toroidal configurations it has been shown that the most static force-free field configurations in a sharp boundary belt pinch geometry are not dynamically accessible via an adiabatic compression of force-free field equilibrium states. This result is independent of stability considerations or the nature of any outer boundaries. The leading asymptotic term (in the ratio column width/column height) for accessible configurations has been determined in the region where poloidal column curvature is small. This term is obtained by requiring closed flux and current surfaces near the sharp boundary of an infinite geometry to move together during an adiabatic compression. If the belt pinch column is a perfectly conducting, uniform, $\beta = 1$ column, then in this region of negligible poloidal curvature, necessary conditions for the accessibility of the equilibrium are that the poloidal field is a decreasing function of poloidal flux, and that toroidal currents in the sheath and neighboring force-free field region are opposed. Moreover, during the compression of a $\beta = 1$ column, the radial motion of both the inner and outer sections of the sharp boundary at the toroidal midplane is in the same direction.

Belt Pinch Free Boundary Equilibria

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Abstract

The plasma shape and location of a perfectly conducting uniform belt pinch column in a container of rectangular cross section with perfectly conducting outer boundary has been determined. The adiabatic compression of the column by a toroidal discharge has been studied. It has been shown that large compression ratios may be obtained only by sacrificing the degree of eccentricity of the column. For compression ratios significantly greater than two, the column cross section becomes nearly circular. The variation of the plasma shape and thermodynamic parameters of the column have been calculated. An essentially explicit solution to the problem has been obtained by conformal mapping.

EFFECT OF MIRRORING ON CONVECTIVE DIFFUSION OF PLASMA

by

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Abstract

Both theory and computer simulation have shown that convective diffusion can be important for systems in which the magnetic field lines are closed. For systems in which the field lines are open and ergodic the electric fields associated with the convective flow are shorted out by charge flow along the field lines. Anything which inhibits this shorting will enhance the importance of convective transport; mirroring is one such mechanism and it occurs in almost all fusion devices. We have developed a two and one-half dimensional particle model which can simulate a plasma with open field lines and with magnetic mirroring.

The particles are charged rods parallel to the z axis and have velocities in the x, y and z directions, however the only electric fields are in the x and y directions. A fixed magnetic field pointing in an arbitrary direction is imposed. A mirror barrier is placed along a line parallel to the y axis. Whether particles pass through the barrier or not is determined by the mirror conditions. This model has been applied to diffusion in a thermal plasma. Preliminary results show that mirroring strongly enhances plasma diffusion as expected. Detailed results of the calculation will be presented.

A Bifurcated Stellarator Equilibrium

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Abstract

A helical equilibrium approximating a theta pinch with low beta in the range of stellarator expansions, is perturbed in a plasma column of finite length. A critical beta is determined and new regions of stability are found by bifurcating this equilibrium at the point of neutral stability. The ideal magnetohydrodynamics free boundary model is employed. The straight helical equilibrium is obtained by expansion in two small parameters, the ratio of the helical magnetic field to the theta pinch field and the product of the helical wave number times the plasma column radius. The distortion of the plasma column from a true cylinder is small of the same order as the ratio of the helical fields to the theta-pinch field, and the helical fields are principally $L = 2$. If the column length is taken small enough, certain "kink instabilities" are avoided and the classical critical beta is obtained. At this beta, a neutrally stable state of principally $m = 1$ long wavelength mode is found, which serves as the starting point of bifurcation. Neighboring bifurcated equilibria are found by expanding the fields in another small parameter. This bifurcated equilibrium is found to be stable when the conducting wall is within 1.8 plasma column radii of the plasma vacuum interface. This position was not sufficient to stabilize the original equilibrium.

Kink Instabilities in Axisymmetric Systems

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Abstract

The symbolic computation technique was continued for the Hill vortex equilibrium solution [1]. The ellipticity was varied up to 10 to 1 and the results are qualitatively in agreement with the cylindrical case. Kink modes are found between integral values of the safety factor $q = \frac{2\pi}{\iota}$. It should be noted that it is easier to find the elliptical cross-section unstable than the circular one.

The poloidal ß was also varied from approximately $\frac{1}{2}$ to 2 and the results remain qualitatively similar.

These results cannot be obtained now for aspect ratios $\lesssim$ 5 probably because our test functions are not yet sophisticated enough.

1 G. Küppers, D. Pfirsch, H. Tasso: Proc. of IAEA Conf. on "Plasma Physics and Nuclear Fusion Research" at Madison I.A.E.A. CN-28/F-14, Vienna 1971

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COMPUTATIONAL STUDIES OF THE TWO-COMPONENT TORUS

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An extensive study of plasma behavior in the two-component torus is made using a variety of theoretical and computational models.

Using a multi-species Fokker-Planck code we compute the factor Q (the ratio of thermonuclear power to injected power) for a variety of reactor conditions, with and without α-particle heating. In these calculations we vary the DT mixture, the injection energy and electron and tritium temperatures. With a two-dimensional, multi-species Fokker-Planck code, where the distributions functions of the ions are functions of energy and pitch angle, we study the effects of beam injection angle and energy clamping.

The equilibrium of the two-component plasma with its confining magnetic field is studied with a two-dimensional equilibrium code. The target plasma is represented by a pressure, either scalar or tensor, which is a function of ψ. The fast ions are simulated by superparticles whose guiding-center motion is computed and which contribute to the toroidal current. With this code we obtain equilibria for various injection angles, energy, and current. The aspect ratio, toroidal field, and plasma current are also varied.

A time-dependent radial transport code has been generalized to include both ion species and neutral beam injection. Particle diffusion and thermal conduction of each species is calculated and both poloidal and toroidal fields vary in time. With this code we study the spatial dependence of beam trapping and slowing down with and without energy clamping.

Three Dimensional Guiding Center Plasmas and Their Instabilities

by Georg Knorr

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Guiding center plasma equations for arbitrary time variable magnetic fields are derived from the Vlasov equation. For a homogeneous, constant magnetic field macroscopic equations are derived. The stability against low frequency ($\omega \ll \Omega_i$) and long wave length ($\lambda \gg r_{\ell,i}$) perturbations in an inhomogeneous plasma are investigated. The electrostatic approximation is being used. It is found for $T_i \ll T_e$ that the two-dimensional shear flow instability of inviscid hydrodynamics persists in three dimensions for positive space charges if the ratio $k_{||}/k_{\perp}$ is not too large. $k_{||}$, $k_{\perp}$ are the wave vectors parallel and perpendicular to B. For negative space charges the instability persists only if $k_{||}/k_{\perp} < v_{\ell,th}/v_d$ which is the ratio of electron thermal velocity over the zero order $E \times B$ drift velocity. A numerical non-linear program in 3D has been written to investigate the development of convective cells known from the 2D case, in three dimensions.

Fluctuations and Cross-Field Transport In Guiding Center Plasma*

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ABSTRACT

A generalization of the Rostoker theory of fluctuations, developed by Williams and Oberman, is applied to the two-dimensional guiding center plasma. Formal expansion in the plasma parameter recovers to lowest order the results of the random phase approximation. This predicts a convergent diffusion coefficient, thus correcting a result of Taylor and Thompson.[1] However, strong wave-wave interactions invalidate the expansion; the random phase approximation is believed to be inadequate for the guiding center model. The electric field autocorrelation is computed. Landau and cyclotron-like damping due to like-unlike and like-like particle pairs compete with diffusive processes as temporal damping mechanisms. The fluctuation spectrum is used to compute the test particle diffusion coefficient.

* Work supported by U. S. AEC Contract AT(11-1)-3073.

† NSF Graduate Fellow

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DISPLACEMENT INSTABILITY OF NUMERICALLY COMPUTED PLASMA EQUILIBRIA

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The stability of numerically computed toroidal equilibria against rigid vertical displacements is studied by computing the net force on the slightly displaced plasma. The computation includes a vacuum region surrounding the plasma and arbitrary flux conserving surface. The work of Laval, et al.,[1] and Rutherford[2] indicates that this mode should be a critical test function giving a necessary condition for stability, even for non-circular toroidal equilibria.

The stability of a large aspect ratio ($R/r = 10$) tokamak with nearly flat current distribution and elliptic cross section ($b/a = 2$), enclosed within a confocal elliptical flux conserving wall, is found to be in excellent agreement with the analytical results of Laval, et al.,[3] for the cylindrical case. For a given cross section toroidal effects have a stabilizing influence.[4] Elliptical cross sections are stable for a finite range of $b/a > 1$.

Plasma cross sections with significant elongation may be stabilized without flux conserving conductors by proper shaping of the plasma surface, e.g., both a rectangular cross section[4] or Ohkawa-type doublet are stable. For some geometries, stability will depend on the plasma current distribution.

It is also found that the stability of a circular tokamak is enhanced by an axisymmetric and vertically symmetric divertor configuration which produces localized field modifications.

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ABSTRACT

THE ION TWO-STREAM INSTABILITY

by

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Recent progress in the development of high current neutral beams by charge exchange justify an examination of the possibility of hot plasma generation by means of the ion two-stream instability. As an example, the injection of an ion beam into a decaying ECRH plasma is considered including effects of a cold electron component. Existing stability theory for a mirror geometry has been modified to include the contribution of the hot ions.

NON-EQUILIBRIUM NEOCLASSICAL DIFFUSION

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A closed set of moment equations is used to describe neoclassical processes in toroidal geometry. In particular, it is shown that the 13 moment equations of Herdan and Liley contain all necessary phenomena. General results are obtained in which classical, Pfirsch-Schluter, plateau and banana regimes are all analytically included. Non-equilibrium situations are characterized by the appearance of surface gradients of the order of r/R and azimuthal and toroidal velocities different from those of an equilibrium situation.

The equations are reduced to a relatively simple form similar to the usual MHD type equations. Both azimuthal and toroidal electric fields are included and the results are essentially independent of aspect ratio and the ratio of poloidal to toroidal magnetic field.

An application to the dynamic unstable mode of a Tokamak discharge is briefly discussed.

Dissipative Universal Instability Due to Trapped Electrons in Toroidal Systems and Anomalous Diffusion*

by

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ABSTRACT

The drift wave instabilities in toroidal systems due to collisions of trapped electrons are analysed including the effects of the sheared magnetic field on both the stability condition and the nonlinear transport. The theoretical predications are checked with the experimental results. The dynamic friction on the fast ion beam due to the drift wave turbulence in a two-component toroidal system (wet wood burner) is also calculated.

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ℓ = 1 EQUILIBRIUM AND STABILITY*

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ABSTRACT

ℓ = 1 helical stability has received continuous study for many years because of its relevance to the Scyllac experiments. All previous work has been based on expansions in two experimental parameters: ϵ, the helical wave number and δ, the helical distortion from a straight theta pinch. Both are normalized to the plasma radius. The final word on sharp boundary theory was J. P. Freidberg's(1) small ϵ, arbitrary δ, numerical theory. Diffuse profile theory has been restricted to one-dimensional, small ϵ and small δ theory.

It is the intent of this calculation to study stability of a straight ℓ = 1 two-dimensional diffuse equilibrium. δ is arbitrary but ϵ is assumed small. The equilibrium is computed numerically and is used in a δW analysis to determine its stability properties.

The equilibrium calculation is a fairly difficult one, involving iteration on several parameters in a non-linear elliptic partial differential equation. Several equilibria have been computed and their stability properties are now being investigated.

* Work performed under the auspices of the U. S. Atomic Energy Commission.

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Correlations of Hard X-Rays and Neutrons with the Parametric Excitation of Waves

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ABSTRACT

Experiments[1] interested in understanding the interaction of a strong electromagnetic radiation (frequency ω_0) with a dense unmagnetized plasma have reported quite intense scattered asymmetric lines with frequency $2\omega_0$ and $\frac{3}{2}\omega_0$. Furthermore, the neutron yield and hard x-ray production are anticorrelated as a function of the peak intensity (focal spot) on the plasma profile (which reaches the overdense region). J. L. Bobin, et al[2], explain the existance of both harmonics only in terms of the parametric ion acoustic instability (photon → plasmon plus phonon), however, this scheme does not explain the anticorrelations.

We show how the observed anticorrelations between the hard x-ray and neutron production can be explained by considering the coupling of the stimulated Brillouin scattering (photon → photon plus phonon) with two instabilities. Namely, the parametric electron-electron decay (photon → plasmon plus plasmon) as a source for the $\frac{3}{2}\omega_0$, and the electron-ion decay (photon → plasmon plus phonon) for the $2\omega_0$. Polarizations, asymmetry of the line, and relative intensities, as well as neutron and hard x-ray output can be discussed within this scheme.

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Abstract Submitted

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March 14, 1974

Non-Linear Rotational Instability of Tokamak Equilibria.* J. E. MC CUNE and K. U. VON HAGENOW, M.I.T. and Max-Planck-Institut für Plasmaphysik, Garching--Self-consistent, finite β, MHD resistive equilibria with finite rotation (inertia) and correspondingly modified mass flux can be found in the Pfirsch-Schlüter regime using Tokamak ordering. It is shown that these 'equilibria' are actually unstable to poloidal spin-up, independently of the rotational velocities existing at the start, in complete analogy with the instability found for zero initial rotation by Hazeltine, Lee, and Rosenbluth in Reference 1. It is speculated that viscous effects would stabilize some of these rotational equilibria, leading to interesting steady states with significant poloidal and toroidal rotation and correspondingly modified diffusion.

*Submitted by J. E. MC CUNE

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Submitted by

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ANALYTIC GUIDING CENTER PLASMA EQUILIBRIA IN THIN TUBULAR MIRROR MAGNETIC WELLS*

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ABSTRACT

We present guiding center equilibria consistent with all of the progressively longer time scales of the guiding center fluid (GCF) model, the particle magnetic drifts, and the Coulomb scattering equilibrium of a neutral injected plasma.

In long thin magnetic wells of Baseball symmetry we show that all particles on a given field line drift on the same surface—we call this an omnigenous surface. Low-β equilibria can then be found analytically by solving magnetic differential equations (MDEs) in polar coordinates (r,θ,z) with arbitrary variations of axial induction $B_o(z)$ and ellipticity $e(z)$ of the flux tube. A typical set of MDEs for J, the longitudinal invariant, is $\nabla\cdot j\vec{B} = B\sqrt{1-\nu B}$, $\nu=\mu/\varepsilon$, $j(z=-1) = 0$; $\nabla\cdot J\vec{B} = 0$, $J(z=+1) = j$. In terms of the magnetic flux function $\psi = B_o r^2 \cosh\eta\ (1+e\cos 2\theta)$, $\tanh\eta=e$, we find $J=J_o(\nu) + J_2(\nu)\psi + 0(r^4)$. An appropriate distribution function for thin tube equilibria is $f = \omega[\psi(J)]\, F(\varepsilon,\mu)$ where $F(\varepsilon,\mu)$ is a velocity space distribution derived analytically or numerically from the Fokker-Planck equation. We show how to calculate the plasma magnetic field from the related drift currents and plasma diamagnetism, thereby completing the equilibrium problem for these cases.

*Work performed under the auspices of the U.S. Atomic Energy Commission

Stability of Drift Time Equilibria without Symmetries

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Abstract

With the exception of the interchange stability condition, none of the previously derived criteria for guiding center stability is valid if the distribution function depends on both flux functions of the magnetic field. (If flux functions can be constructed such that the distribution function depends only on one of them, there is a particular family of surfaces which are independent of the velocity space variables, and which are called "generalized pressure surfaces" because they are pressure surfaces if the pressure is isotropic.) However, drift time equilibria (the distribution function depends on the flux functions only through the longitudinal adiabatic invariant) do not possess generalized pressure surfaces except in special cases such as an ignorable coordinate. Hence to determine the stability of drift time equilibria in current experiments it is necessary to allow for more general distribution functions than previously considered. Necessary conditions for the stability of such equilibria (with nonergodic field lines) are derived from the energy principle by considering two special classes of perturbations: line preserving perturbations, and a class of localized perturbations. These conditions require the positivity (at each magnetic field line) of certain 2×2 matrices of equilibrium quantities (similar to the matrix appearing in the well known interchange criterion for drift time equilibria). Although these matrices have not been analyzed in detail, it appears that most of the features of the simple case with generalized pressure surfaces carry over to the general situation. Like the interchange criterion, these necessary conditions do not apply to line-tied equilibria because in these the displacements considered are not admissible.

FOCAL - POINT INSTABILITIES: AN APPARENT COUNTEREXAMPLE TO THE MHD ENERGY PRINCIPLE*

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ABSTRACT

Examples are given of so-called "focal-point instabilities" in an energetically stable MHD system. The focal point referred to is a null point of the equilibrium $\underline{B}$ field, at which a local concentration of perturbation energy occurs, with the result that the local energy density (evaluated at that one single point) shows an exponential growth. The unstable modes have a vanishing norm, however, and thus are not included in the standard MHD energy criterion.

*Work performed under the auspices of the U.S. Atomic Energy Commission

AN INDUCTIVE MODEL FOR MULTIDIMENSIONAL PLASMA SIMULATION*

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The non-radiative limit of the Maxwell-Lorentz system was first given by Darwin (1). A numerical formulation of this model in one space dimension has been described by Haber et. al. (2) and similar algorithms have been widely used elsewhere. However, their method does not appear to generalize to more space dimensions. We have devised a method which applies in two and three dimensions and which has advantages in one space dimension as well. The electrostatic and magnetic fields are obtained by straightforward direct solutions of

$$\nabla^2\varphi = -4\pi\rho \quad \text{and} \quad \nabla^2 B = -(4\pi/c)\nabla x J \ .$$

The transverse or divergence free part of the electric field is determined by the equation

$$\nabla^2 E_t = (4\pi/c^2)\ \dot{J} - (1/c^2)\ \nabla\ddot{\varphi}\ .$$

The time derivative of the current density is eliminated by the simulation equivalent of the Vlasov equation leading to an equation

$$\nabla^2 E_t + \eta E_t = S - (1/c^2)\ \nabla\ddot{\varphi}$$

where η and S are defined by moments over the simulation ensemble. Together with $\nabla \cdot E_t = 0$ this determines E_t and $\ddot{\varphi}$ simultaneously. In one space dimension this system decouples completely and direct solution is immediate. In two dimensions we have developed a global iteration technique (3) suited to these equations. As a test problem we have applied the method to the Weibel instability in two dimensions.

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* Work was performed under the auspices of the U.S.A.E.C.

Large Amplitude Convective Cells and the Low-Frequency Electrostatic Instabilities*

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ABSTRACT

Convective plasma diffusion due to thermal vortices $(\omega = -ik_\perp^2\mu_\perp)$ in a uniform plasma is considerably reduced in an inhomogeneous plasma because the fluctuation energy at $\omega = -ik_\perp^2\mu_i$ shifts to ion flute mode at $\omega = \omega^*$ from convective cells $(\omega^*/k_\perp^2\mu_i \gtrsim 10^5$ for $n = 10^{14}cm^{-3}$, $T = 1$ keV, $B = 50$ kG, $k_\perp\rho_i \lesssim 1)$.[1] However, since the net charge or the electric field associated with the stable drift oscillation is quite small $[(E^2)k/8\pi = T/2(1 + \omega_{pi}^2/\Omega_i^2)]$, it appears that quite a modest amount of superthermal charge imbalance across density gradient and magnetic field will dominate drift oscillation, thereby causing a large diffusion. One sufficient condition for this to occur will be $k^2D_\perp \gtrsim \omega^*$ where $D_\perp$ is the particle diffusion due to superthermal cells. It is possible that a superthermal cell does not see the gradient, then the condition will be much weaker. Results of 2D simulation with superthermal convective cells excited across B and the gradient show an enhanced diffusion regardless of the gradients, indicating weak dependence on $(E^2)k$ and B of $D_\perp$.

Superthermal convective cells and enhanced transports appear inherent to magnetically confined plasmas due to various causes such as plasma formation and low-frequency electrostatic instabilities (drift, flute, trapped-particle) which build up charge imbalance across B and ∇n. In order to study the formation of convective cells, saturation amplitude and particle diffusion associated with the low-frequency instabilities, we have simulated a current-driven, low-frequency drift instability. The most unstable mode saturates at $(E^2)k/4\pi T \sim 50$ with $D_\perp \sim 1000\ D_\perp^{classical}$. Saturation level seems to be low $(E^2/4\pi nT \lesssim 10^{-3})$ and the dominant saturation mechanism appears the disappearance of gradient (change of distribution) rather than orbit diffusion for strongly unstable case $(\gamma_k \lesssim \omega_k)$.

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Probability Distributions for the Electric Field in Weak Turbulence Theory*

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Weak turbulence theory is incomplete, in the sense that only the ensemble average of the fields is determined. A complete theory should determine the probability distributions for the fields, including joint distributions for various Fourier components of the field and multiple time distributions.

The functional form of all such distributions is determined using the central limit theorem of probability theory and two assumptions within the spirit of weak turbulence theory. This general theory is applied to the special case where the turbulence is dominated by the wave particle interaction. For this case quantities entering the distributions as parameters, such as the mean and standard deviation, are determined by quasi-linear theory and Dupree's recent theory of phase space granulation, or "clumps."

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Numerical and Theoretical investigations on a reverse field pinch experiment at Padua University

Sergio Ortolani

The purpose of this paper is to shortly report some numerical and theoretical investigations related to the scientific aims of the toroidal pinch experiment next to be operating in Padua. This experiment, described earlier [1], will at the beginning work in the reverse field pinch configuration (RFP). This configuration has been experimentally analysed during the past years [2, 3] and, due to its good stability chances, is extensively studied in present day experiments (mainly at Culham and Los Alamos Laboratories).

Experimental evidence of the possibility to set up a stable RFP which decays with the classical resistive time have been obtained in the Culham HBTXI experiment [4, 5] only in the low (~ 10 eV) temperature regime; in this experiment the relatively long rise time (several μs) of the main fields can limit the maximum temperature compatible with stability. In the Los Alamos experiment ZT-1, shock heating by the magnetic field of a very fast rising Z-current (100'nsec) is used to give high temperatures plasmas [6], but fully stable profiles have not jet been obtained. Then open questions deal with the possibility to set up stable RFP configurations at higher temperatures.

The Padua experiment has, in a first attempt, the goal of investigating, the regime of faster magnetic field rise times (~ 1 μs) than in the Culham experiment, in the hope of producing higher temperatures (up to 100 eV) compatible with stability.

In this report we want just mention some results concerning the MHD stability of RFP configurations, the numerical simulation by a 1D-MHD code of the working regime of the Padua experiment and a preliminary study of the possible role of small traces of impurity atoms.

A set of necessary conditions for the ideal MHD stability of reverse field plasma profiles has been derived by Robinson [7] on the basis of the energy principle.

The relevant conditions for stability concern the magnetic field pitch profile, the value of the poloidal beta, β_θ, which must be below a critical value, and the total axial magnetic flux which must not reverse.

The proximity to sufficiency of such necessary conditions has been strengthened by experimental evidences [5] and by numerical calculations [8], especially with regard to the flux condition. In particular the normal mode stability analysis of several RFP profiles [9] shew as the flux condition is a very critical one.

As an example of theese results in figure 1 is reported the growth rate spectrum (m=1) for three profiles which are characterized by the value of the total axial flux: case I corresponds to a slightly positive flux, case II has an approximately zero total flux and case III corresponds to a slightly negative flux. We see as the maximum growth rate strongly increases as the flux becomes slightly negative. Moreover only the long wavelength part of the spectrum corresponds to real gross m=1 modes, while the large k part corresponds to extremely localized Suydam's modes which are expected to be smoothed by finite larmor radius effects.

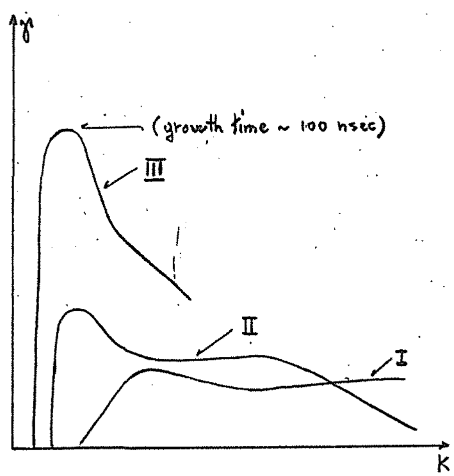

FIG. 1

Moreover a series of MHD computations enphasized that, in practical terms, the flux condition is the more restrictive one in the sense that if it is satisfied, the others conditions are easily (if not consequently) satisfied.

In the light of the previous considerations a set of numerical computations by a one-dimension, time dependent MHD code (developed at Culham), including circuits equations to couple plasma and external circuits, have been done in order to pick out a working regime for the experiment suitable to set up MHD stable configurations.

One of the more significant results is that obtained by varying the amplitudes of the bias axial magnetic field and of

the fast B_z-field (i.e. the capacitor voltage for the I_θ-circuit, $V_{I_\theta\text{-circuit}}$). For example with a filling pressure of 40 mtorr and a feeding energy for the $I_{z\text{-circuit}}$ of 14.6 Kjoules (which leads to a plasma current of about 120 KA) we get the results shown in figure 2. We see as the range of the useful working conditions (unshaded region) is rather narrow. On the other hand by rearranging the distribution of the energy into the different circuits (i.e. by enhancing the feeding energy for the Iz-circuit to get a plasma current of about 200 KA) a better utilization of the bank become possible. For this last case computed average temperatures of about $\overline{T}=\overline{T}_e+\overline{T}_i \simeq 80$ eV, to be compared with $\overline{T} \simeq 50$ eV for the situation of figure 2, become compatible with the stability requirements.

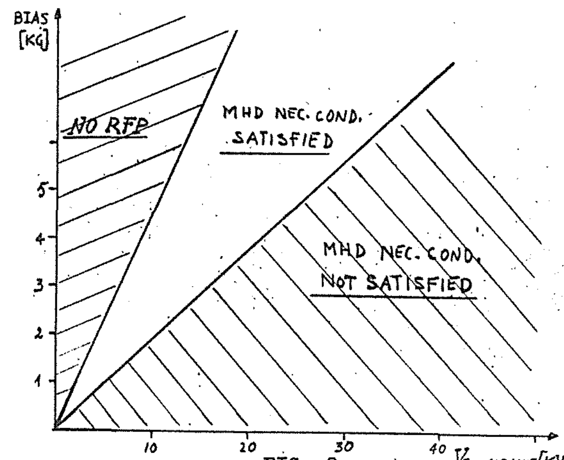

FIG. 2

A first attempt to investigate the diffusion phase of RFP profiles was also done by using the crow-bar facility included in the circuits routines of the MHD code (we mention here that long computer time makes this kind of utilization wasteful and the use of an ad hoc developed computer program is foreseen for the diffusion phase).

Any way our few results seems to confirm the observed increase in the poloidal beta [10]. This is an expected trend of ohmic heated discharges [11] and can lead to instability by exceeding the critical β_θ (0.6÷0.8).

The behaviour of the diffusion phase is of course related to the balance between the different heating and loss mechanisms. The energy loss by impurities line radiation (not included in the MHD code), which can be catastrophic in a shock-heated discharge unless the temperature reached after the fast implosion is high enough, can be important also in ohmic heated discharges even if the effect of impuri rities on the energy balance is partially masked by the enhanced resistivity. By solving the time dependent problem to determine the population of the ionization stages of impurity species (in the limit of the corona model) computations are presently being developed to understand the role of small traces of impurities in the energy balance for the range of parameters of the Padua experiment.

- report prepared by S.Ortolani -

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Guiding Center Screw Pinch Spectrum and Stability

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Abstract

Instabilities and their growth rates for a linear screw pinch are studied by using the normal mode analysis based on the lowest order guiding center equations of motion. The problem can be reduced to a single second order ordinary differential equation, the normal mode equation, which is similar to the Hain-Lüst equation in MHD except that its coefficients contain Cauchy-type integrals of the equilibrium distribution function.

The normal mode equation should serve as a basis for analytic and numerical studies on the spectrum and stability of the diffuse pinch. In a first attempt to draw some analytic results, the equation is used to examine the line-preserving instabilities for no-shear systems, the semi-local instabilities for low-shear systems and the Suydam-type local instabilities. The growth rates and eigenfunctions for these instabilities are derived. It is found that for low-shear systems the maximum growth rate is given by the zero-shear line-preserving modes. Comparison with MHD results is discussed in terms of thresholds and growth rates for these instabilities.

ON THE ROLE OF SUPRATHERMAL FLUCTUATIONS ON THE ENERGETIC IONS IN A TWO-COMPONENT WET WOOD BURNER PLASMA

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Target plasmas used for the production of thermonuclear energy in the wet wood burner (two-component reactor) scheme are likely to be far from thermodynamic equilibrium having strong currents and $T_e > T_i$. In such plasmas the fluctuation spectrum may be enhanced to the point where slowing down and scattering of energetic injected ions from stable but non-thermal waves becomes important. In this paper we present a discussion with numerical evaluations of the relative importance of the collisional and collisionless Fokker-Planck coefficients as a function of T_e/T_i, V_{de}/C_s and V_T (the velocity of the test ions) and we indicate regimes in parameter space for the case of ion waves where the neglect of the wave contributions can lead to inaccurate results. Extension of the results to dispersion relations governing the fully magnetized plasma, to non-parallel injection, and to generalized Fokker-Planck models for computation will be discussed.

On Radio Frequency Heating of Tokamaks*

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ABSTRACT

Two results are presented: (1) calculations of the nonlinear effects of heating by magnetosonic waves in tokamaks and other devices, and (2) the damping of lower-hybrid waves in tokamak geometries. Three types of nonlinear processes are induced by magnetosonic waves: (1) velocity space diffusion, (2) real space diffusion, and (3) parametric instabilities. We will present quasilinear equations giving the velocity space diffusion, argue that real space diffusion is negligable, and identify a parametric instability which occurs for pump frequencies in the ion-cyclotron frequency range. The damping of lower-hybrid waves in a plasma with varying magnetic field intensity (so that there are many regions where $\omega_{pi} \sim \omega = k_{\parallel}v_{\parallel} + n\omega_{ci}$) is shown to be the same as the Landau damping by unmagnetized ions.

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A Waterbag Simulation of the Tokamak

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For the interesting experimental case of aspect ratios approaching unity, conventional theory of toroidal plasmas is inapplicable. A simulation of the collisional fluid equations in the poloidal plane has therefore been developed. One-dimensional theory and simulations have shown the occurrence of a skin-effect in contradiction with experiment. Results from this two-dimensional simulation show the creation of a dipole structure during the formation of the pinch which considerably enhances the diffusion of the poloidal magnetic field and leads to the formation of a contained toroidal column in times of 1 millisecond. The dipole structure is produced by enhanced currents and temperatures on the inside of the torus, while rapid diffusion of the field occurs in the colder plasma on the outside of the torus. A neutral point is formed which considerably enhances the resistive destruction of the field. Temperature gradients around the flux surfaces are maintained due to the limitation of the heat conductivity by large "q-values" during the formation stages.

The simulation is executed in the time-dependent flux-surface co-ordinates. The orthogonal co-ordinates to the flux surfaces are reconstructed at each step. The simulation is thus both Lagrangian and orthogonal. These new techniques developed are powerful in dealing with multidimensional fluid problems and would appear essential in simulating high-temperature, magnetically-contained plasmas.

COMPUTATIONAL STUDIES OF MIRROR CONFINEMENT*

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ABSTRACT

Plasma confinement in mirror systems is studied via the two-dimensional velocity space Fokker-Planck equations for ions and electrons. This model is used to analyze both the BB II and 2X II experiments as well as more general aspects of mirror confinement with neutral beam injection.

From the ion and electron distributions the density, $n(B)$, pressure tensor, $\underline{\underline{P}}(B)$, and ambipolar potential, $\phi(B)$, are obtained as functions of magnetic field strength, B, along a flux tube. The MHD stability conditions for firehose and mirror modes are then used to find the limiting value of β for given mirror ratio and neutral injection source. The beam injection angle relative to field lines is an important parameter in determining the maximum stable beta.

The actual spatial dependence of the plasma density and pressure along a flux tube is deduced from $n(B)$ and $\underline{\underline{P}}(B)$ if the vacuum field spatial profile is known and if one assumes a long, thin plasma so that $B_{vac}^2 = B^2 + 8\pi\, p_{\perp}(B)$. Profiles are calculated for several values of β between zero and the stability limit.

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PARTICLE ORBIT LOSS-CONES IN TOKAMAKS*

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In present day tokamaks, energetic injected ions that are barely trapped have banana widths which are of the order of the plasma minor radius. These particles may hit the limiter or be charge-exchanged in the high neutral density which exists near the limiter. This loss of particles constitutes a "loss-cone" and may be important even though the injected ions are produced on well-contained orbits.

Solution of the Fokker-Planck equation shows that as injected ions slow down via coulomb collisions with the background plasma, they also scatter in pitch angle. This scattering is greatly enhanced by impurities in the plasma. Because of this, injected ions have a reasonable probability of scattering into the loss-cone.

From particle drift calculations, we find that the loss cone is centered at a pitch angle greater than 90°. Thus, counterinjected ions lie much closer to the loss-cone than do coinjected ions and hence will be preferentially lost. The perpendicular charge exchange spectrum in tokamaks can be used to verify the existence of this loss-cone. Specifically, the charge exchange flux from the counterinjected beam should go to zero abruptly above the minimum energy for which the loss-cone occurs. No such loss should occur for coinjected ions because the observed ions would not yet have reached the loss-cone since it occurs only at angles greater than 90°. Preliminary charge-exchange measurements on ORMAK and ATC seem to indicate the presence of this effect.

Only a fraction of the "loss-cone" ions will hit the limiter since they will probably undergo charge exchange before they reach the limiter radius. This might give rise to sputtering problems at the wall.

Finally, we note that the preferential loss of the counter-injected ions makes it more difficult to cancel out the beam associated currents, momenta, etc.[1] that may be deleterious to the plasma. Also, this loss-cone must be taken into account when calculating the heating due to injection. The magnitude of the latter effect will be discussed.

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*Research sponsored by the U. S. Atomic Energy Commission under contract with Union Carbide Corporation.
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∓On leave of absence from M. I. T.
[1]J. D. Callen, J. F. Clarke, J. A. Rome, "The Theory of Neutral Beam Injection into a Tokamak," Paper E14, Third Int'l. Symposium on Toroidal Plasma Confinement, Garching (1973).

Uniform Stellarator to Scyllac Expansion

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Abstract

An expansion covering the region between the areas of stability of the scyllac and stellarator scalings is presented.

In the scyllac case, for example, a combination of small $kr^{(0)}$, $\ell = 1$ and large $\beta(\sim 0.7)$ gives stability; here, $r^{(0)}$ is the radius of the plasma, $k = \frac{2\pi}{\lambda}$ (λ is the wave-length of the helical winding), ℓ is the order of the winding and β is the ratio of plasma to magnetic pressure. In the stellarator case there exists a range of β up to a critical β^* for which stability is achieved. However, neither of these sets of results is obtainable by starting from the other scaling; it is the purpose of this paper to provide the transition from one to the other.

We consider the stability of a straight, cylindrical plasma column of length L, radius $r^{(0)}$ and constant pressure p and carrying a constant magnetic field B_i parallel to the axis in addition to a small helical field. A constant magnetic field B_o parallel to B_i is applied to the vacuum outside the plasma. The stability of the configuration is studied by use of an energy principle in which the relevant parameters are ε, β, ℓ and $\chi^{(0)}$. Here ε is the ratio of the helical to the applied field inside the plasma and $\chi^{(0)} = kr^{(0)}$. The energy integral δW, expanded in powers of ε takes the form $\delta W \sim \varepsilon^2 f(\beta) + \varepsilon^4 g(\beta)$ where β is arbitrary. For small values of $\chi^{(0)}$ we can show that $f(\beta) \sim a\beta + b\beta^2 + \cdots$. For any $\chi^{(0)}$, $g(\beta)$ has the form $g(\beta) \sim 1 + c\beta + \cdots$. The critical β, obtained from $\delta W = 0$ is $\beta = \beta_c \sim \varepsilon^2$ on the stellarator scaling. Keeping β arbitrary leads to the well-known scyllac results.

The interesting results are the full transitions from one stable region to the other for various values of the parameter β ($\chi^{(0)}$ and ℓ may also be varied). These are in the process of evaluation using Macsyma.

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Stability of an Ion Beam To Velocity Space Instabilities*

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ABSTRACT

The steady state distribution of an ion beam injected nearly parallel to the magnetic field and slowing down in a background plasma has been investigated for possible velocity space instabilities. The beam is a small fraction of the overall plasma density, and hence, stability can only be produced from inverse dissipation arising from the resonances at $V_{\|} = (\omega - n\omega_{ci})/k_{\|}$. For long wavelengths, the dominant instability term comes from the n = -1 resonance, but it is difficult to find particles that resonate at this velocity. For example, waves corresponding to the fast Alfven-Whistler branch would resonate a minimum particle velocity $V_{\|} = 1.5(3)^{1/2} V_A$, (where V_A is the Alfven speed), which corresponds to an injection energy far higher than has been contemplated. The n = 0 resonance can cause instability for the low-frequency fast Alfven wave, if $V_A < V_{\|}$, a condition that may at times be satisfied in two component systems. This is a weak instability, and the quasi-linear theory predicts the relaxation of the distribution to one that damps this mode. The shear Alfven wave will not be destabilized by the n = 0 resonance because the wave polarization produces an extremely small matrix element that is dominated by electron Landau damping. Similarly, any destabilizing tendency of loss cone like resonances tend to be stabilized by electron Landau damping.

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NON-LINEAR KINK AND TEARING MODES IN TOKOMAKS*

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ABSTRACT

The non-linear unstable behavior of low β kink modes of given helical symmetry in Tokomaks is investigated. Results with uniform plasma current density show widely differing types of behavior, simple bifurcation to neighboring helical equilibria; very complicated and distorted elastic motions, and bubble formation, depending on how close the q value is to integral. In cases where the formation of a vacuum bubble is energetically favorable, the accessibility of this final state is examined.

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<u>Inhomogeneous Threshold in Beam-Plasma Systems</u>

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The instability produced by a relativistic (or nonrelativistic) weak beam in a plasma is due to resonant coupling between the beam and a plasma eigenmode. In a plasma with gradients in the direction of beam propagation, a detuning of this resonance takes place in a manner similar to inhomogeneous detuning of parametric instabilities. If the plasma nonuniformity is sufficiently large the instability is quenched. The quenching condition is $\lambda \equiv \left| \frac{\beta}{\alpha} \left(\frac{V^2 \alpha}{\omega_b^2} \right)^{2/3} \right| \gtrsim 1$, where $\beta = - \frac{\partial \varepsilon_p}{\partial x}$, $\alpha = - \frac{\partial \varepsilon_p}{\partial k}$, V is the beam velocity and ω_b is the beam plasma frequency multiplied by $\gamma_b^{-3/2}$ for a relativistic beam. Consequently in a beam heated plasma energy is delivered in the plasma interior, not at the surface.

The third order differential equation describing the beam plasma interaction may be solved on the Fourier transformed k plane, and asymptotic solutions for the wave amplitude as a function of x can be found. These are in good agreement with computer solutions. It is interesting to note that the plasma mode, essentially uncoupled from the beam far away from the resonance region converts into a beam mode on the other side of the resonance. The consequences of including nonlinearities and finite beam spread are qualitatively discussed.

Abstract Submitted

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of Controlled Thermonuclear Research

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<u>Ion Distribution Function in Near Collisionless Tokamak Plasmas</u>.* D. J. SIGMAR and S. P. HIRSHMAN, <u>M.I.T.</u> --Spectroscopic measurements of the H_α^* line from charge exchange provides information on the velocity distribution function of hot ions.[1] In particular, the line shape obtained from observation in the tangential direction is related to $F_p(v_\parallel) = \int 2\pi v_\perp dv_\perp f_p$, where f_p is the proton distribution function. A comparison of the experimental and theoretical shape of $F_p(v_\parallel)$ can be used to determine the existence of proton banana effects in a given experiment.[1] Following neoclassical theory[2,3] we calculate $F_p(v_\parallel)$ to $O(\sqrt{\varepsilon})$, in the banana- and the plateau-regime. In the banana regime, the result is a shifted Maxwellian with the parallel drift velocity given by the diamagnetic response to the ion pressure gradient and the ambipolar electric field, plus significant odd distortions of $O(\sqrt{\varepsilon})$ for superthermal particles, driven by the temperature gradient. Significantly for present experiments in the plateau regime, the shape of $F_p(v_\parallel)$ remains similar with certain constants becoming functions of the collisionality parameter. Quantitative agreement with experiments will be mentioned briefly.

*Submitted by D. J. SIGMAR

Research performed under USAEC Contract AT(11-1)3070

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[3]F. L. Hinton and M. N. Rosenbluth, Phys. Fluids, <u>16</u>, 6 (1973).

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Interchange Growth Rates

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Abstract

The well-developed variational methods of conventional δW analysis can be used to obtain information about the maximum MHD growth rate σ_{max} without solving the eigenvalue problem. Since σ^2_{max} is the smallest number for which $\delta W_\sigma = \delta W + \sigma^2 K \geq 0$, sufficient conditions for $\delta W_\sigma \geq 0$ yield upper bounds for σ_{max}, while necessary conditions yield lower bounds. As a particularly simple application, a lower bound is obtained in arbitrary equilibria with closed magnetic field lines by considering only interchange test functions in δW_σ. The result can be written as $\sigma^2_{max} \geq f^2 I/\rho$, where ρ is the mass density, f is some quantity depending on the geometry of the equilibrium (f = 1 in a straight circular cylinder), and $\gamma I = \nabla q^{-\gamma} \cdot \nabla p q^\gamma$, $q = \int d\ell/B$ ($I \leq 0$ is the well known interchange stability criterion). This bound should be still valid in equilibria with small shear (at arbitrary β), thus being relevant for a variety of experiments. At low β, it should asymptotically approach the maximum growth rate itself. (In a low β low shear screw pinch this can be verified by comparing with the result of the so called semilocal analysis). Quantitative calculations are performed for bumpy θ pinches in order to find out how accurate the present estimate is in high β equilibria of practical interest for which growth rates have been numerically obtained in the past.

Magnetic Wells and Average Magnetic Wells

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Abstract

For guiding center distributions $f(\varepsilon,\mu,\psi)$ $(\frac{\partial f}{\partial \varepsilon} \leq 0)$ in a magnetic field $\underset{\sim}{B} = \nabla\psi \times \nabla\theta$ with nonergodic field lines, a suff icient stability condition,* when expanded to the lowest significant order in β, becomes

$$\int \frac{d\ell}{B^2} \frac{\partial p_\perp(\psi,B)}{\partial\psi} \quad \frac{\partial B(\psi,\theta,\ell)}{\partial\psi} < 0 \ ,$$

Although this inequality is not new insofar as it is equivalent to a well known sufficient condition for interchange stability, its novel feature of guaranteeing stability to arbitrary perturbations motivates a closer look at it: Stability obtains if the increase of B across the flux surfaces is opposite to that of $p_\perp$ in an average sense, and an average outward increase of B is favorable for stable confinement. The quantitative meaning of this qualitative statement depends on the distribution function to be confined. For $f(\varepsilon,\psi)$, $p_\perp = p_\parallel = p(\psi)$, and the stability condition reduces to $\frac{dp}{d\psi}\frac{dq}{d\psi} > 0$ $(q = \int d\ell/B)$, so that "maximum q" (often called "average magnetic well") is favorable. For $f(\varepsilon,\mu)$, one has $p_\perp(B)$, and the stability condition is marginal, thus not providing any conclusion. But since $f(\varepsilon,\mu)$ is known to be automatically stable, there is only a confinement condition, requiring "maximum B" (often called "magnetic well"). The present sufficient stability condition replaces the unnecessarily restrictive magnetic well requirement by an extension to tensor pressure of the average magnetic well requirement of the scalar pressure theory.

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* G. O. Spies and D. B. Nelson, to appear.

ION CYCLOTRON WAVES AND FAST WAVES IN A TOROIDAL CAVITY

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ABSTRACT

We investigate the propagation of ion cyclotron waves and fast waves in a toroidal cavity with a spatially varying static magnetic field as in a tokamak or stellarator. Using a cold plasma model in a square cross-section conducting toroidal cavity and neglecting effects of electron inertia, we reduce the wave equation to an ordinary differential equation which is numerically integrated subject to boundary conditions to find eigenmodes which represent cavity resonances. For the fast wave, we find only very small perturbations from the corresponding uniform plasma guide problem and conclude effects of toroidicity on the fast wave are not significant. For the ion cyclotron wave we find the region of propagation is generally confined to the high magnetic field side of the cyclotron resonance and for low order modes vertically and azimuthally, the region or propagation does not extend to the resonance location. A new type of wave behavior appears due to toroidicity which is similar to the ion cyclotron wave but depends differently on wavelength and may be important for fusion plasma heating because of its possibly greater accessibility. On the basis of the wave profiles, optimum antenna designs will be profoundly affected by toroidicity.

Tokamak Heating by MHD Resonance Phenomena

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Abstract

The dynamical behavior of certain plasma configurations, such as the sheet pinch and the screw pinch, when described by the linearized equations of ideal MHD, is characterized by a spectrum which can have both discrete and continuous components. The continua are a consequence of non-square integrable spatial singularities of the eigenfunctions. An implication of the continua, which has already been considered in the past for the sheet pinch geometry[1], is local absorption of rf energy. In this paper this absorption process is examined for the screw pinch geometry with equilibrium profiles similar to those in current Tokamak experiments. The singularities are first identified and then related to the resonances and cutoffs of the slow, fast and cusp Alfven MHD waves in an infinite uniform plasma. For the proposed external source considered here, it is found that in the screw pinch, local resonances exist only in the cusp and Alfven wave modes, while the fast wave mode leads to a point of local reflection of energy which corresponds to a cutoff. Connection formulae across the spatial singularities at the local resonances are derived and then used to obtain an expression giving the rate at which energy is transferred from an external sheet current to the various resonances. The dependence of the power absorbed by the plasma on the applied frequence is examined both analytically and numerically. A particular set of profiles which is examined is that which is found in the ST Tokamak. Energy transfer to these profiles as a function of the plasma parameters is calculated. Preliminary results obtained from a sheet pinch model show that the resistive part of the coil impedence at interesting experimental frequencies are in the order of ohms. Comparisons of energy absorption with other methods, i.e., lower hybrid resonance schemes, are made.

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Closed Line Plasma Equilibria

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Abstract

A general theory of plasma equilibria with closed magnetic field lines has been developed for the guiding center model.[1] The theory, which is subject to mirror reflection symmetry, is not restricted to an ignorable coordinate and is therefore applicable to bumpy tori. Both the appropriate analytical methods and the ensuing plasma properties differ profoundly for magnetic surface and closed line equilibria.

Under the appropriate conditions of marginal equilibrium[1], which is distinguished by the eigenvalue structure of the equilibrium equations, the pressure surface topology is not simple and pressure islands develop. This is distinct from magnetic islands since the magnetic lines remain closed with zero rotational transform. The development of these pressure islands due to a marginal perturbation of the equilibrium is established in the very singular, low β shallow-well approximation[2]. In this limit explicit criteria[3] indicating the appearance of the non-simple pressure contours are extended for specific pressure profiles. The toroidal perturbation and the effect of a vertical field, which in this case do not induce a simple radial shift, are calculated. Although the pressure islands are found to have a complex periodic structure, estimates of their thickness are obtained.

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2 J. B. Taylor and R. J. Hastie, Phys. Fluids 8, 323 (1965).

3 V. L. Teofilo, Bull. Am. Phys. Soc. 18, 1353 (1973).

Plasma Heating by Intense Relativistic Electron Beams With Return Current*

By

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ABSTRACT

We have performed a series of one-dimensional simulations of the relativistic two-stream interaction with a return current present. The return current is modeled by generating an electric field which keeps the net current zero. The Buneman instability is stabilized by a combination of plasma electron trapping and nonlinear scattering off thermal electrons. Bulk heating of the electrons is observed, whereas energetic tails (6keV) are produced on the ion distribution. The two-stream instability is stabilized by beam electron trapping in agreement with an analytical model. With a beam present additional plasma heating takes place with maximum ion energies up to 10keV being observed. Electron heating is found to be proportional to the initial beam energy; however, the ion heating is relatively independent of initial beam energy. The induced electric field acts as a drag on the beam and accounts for approximately 10% of the beam energy loss in these runs. The turbulent collision frequency needed to estimate the decay time of the return current is found to be $\nu^{*} \cong 0.2 \sim 0.4\omega_i$ where ω_i is the ion plasma frequency. The turbulent energy spectrum scales like $|E_k|^2 \sim k^{-2}$.

* Work supported by the Atomic Energy Commission and the Office of Naval Research.

Resonances and Plasma Heating*

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We discuss the role of low frequency resonances in the r.f. heating of plasmas. For simplicity consider a slab geometry, first in the ideal M.H.D. approximation. The displacement $\xi_x(\omega,k,z)\ \exp\ i(\omega t-k_\perp \cdot x_\perp)$ satisfies

$$\frac{d}{dx}\left[\rho\ C_T\ \frac{(\omega^2\ \ \omega_o^2)(\omega^2\ \ \omega_2^2)}{(\omega^2\ \ \omega_1^2)(\omega^2\ \ \omega_3^2)}\ \frac{d\xi}{dx}\right] + \rho(\omega^2-\omega_2^2)\xi = 0$$

where $\omega_o^2 = \frac{C_A^2 C_S^2 k_\perp^2}{C_T^2 k^2}$; $\omega_2^2 = C_A^2 k_\parallel^2$,

$$\omega_{\substack{1\\2}}^2 = \frac{1}{2}\ C_T^2 k^2 \left[1 \mp \sqrt{1 - 4\omega_o^2/C_T^2 k^2}\ \right]$$

$C_T^2 = C_A^2 + C_S^2$, C_A and C_S being the Alfven and sound speeds.

The solutions have singularities at $\omega = \omega_o$, $\omega = \omega_2$, and for a single wave, appear to display anomalously large power absorption.(1) Consideration of a wave packet shows, however, that the singularity is almost cancelled by an internal Alfven wave, and certain paradoxes are clarified. W.K.B. solutions are given and the power absorption estimated.

Using the Vlasov equation and the small Larmor radius expansion, the field line displacement satisfies

$$\frac{d}{dx}\left[\frac{\rho(C_S^2\alpha+C_A^2)(\omega^2-C_A^2 k_\parallel^2)}{\omega^2-C_A^2k^2-k_y^2\alpha\ C_S^2}\ \frac{d\xi}{dx}\right] - \rho(\omega^2-C_A^2k_\parallel^2)\xi = 0$$

where $\alpha(\omega,k_\parallel)$ is a complex function involving trajectory integrals, which becomes large where the condition for the propagation of the ion acoustic mode is satisfied. Many of the properties of the M.H.D. waves are reflected in these plasma waves, which are again handled in the W.K.B. approximation.

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FOKKER-PLANCK EQUATION FOR A PLASMA IN A MAGNETIC FIELD*

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ABSTRACT

A convergent Fokker-Planck equation is derived for a one-species plasma in a strong dc magnetic field. Conservation laws and an H-theorem are proved. The equation is manipulated into a form in which it is straightforward to evaluate magnetic field dependent relaxation times in terms of series of modified Bessel functions.

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SPECTRA AND GROWTH RATES OF A GENERALIZED SCREW PINCH

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Abstract

The theory of Weitzner[1] for the bumpy θ pinch is extended to the case of a generalized screw pinch in which the equilibrium magnetic field is $\underset{\sim}{B} = (\varepsilon B_r, \varepsilon B_\theta, B_z)$. ε represents the slow periodic z-variation of the axisymmetric equilibrium and the stability relative to their r-variation. Perturbation expansions in ε and in the bumpiness of the flux surfaces are performed on the linearized equations of ideal magnetohydrodynamics. From the theory of Grad[2] there should be four stable continua-two of which are found for modes with $\omega^2 = O(\varepsilon^2)$, while for the transverse modes a generalized Suydam criterion for local instability is derived. The (global) growth rates of the most unstable transverse modes are computed numerically for various B_θ and the results are compared to those of the bumpy θ pinch (where $B_\theta \equiv 0$) as well as to the ordinary screw pinch (where $B_r \equiv 0$).

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[2] H. Grad, Prod. Nat. Acad. Sci., USA, 70, 3277 (1973).

LARGE ASPECT HIGH β TOKAMAK STABILITY

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Abstract

Various axisymmetric diffuse profiles are considered for a Tokamak for ranges $\beta \sim 1$ to $\varepsilon^{2/3}$ and $\beta_p \sim 1$ to $\varepsilon^{-4/3}$ where ε is the inverse aspect ratio. The double expansion technique employed in the bumpy θ pinch[1] and bumpy screw pinch[2] is again used, assuming small deviations from circular cross sections. In all cases the stability is governed by that of the straight cylindrical screw pinch (previously investigated[2] for a tokamak- and scyllac-like profile, in which the stable Alfven continuum is pushed[3] to the unstable side emitting an infinite number of discrete modes). The plasma pressure is supported by the toroidal magnetic field (except for the case $\beta \sim 1$, $\beta_p \sim 1$ in which the pressure is supported by both toroidal and poloidal fields). There is no mode-mode coupling - in contrast to the bumpy (straight) pinch results[1,2] where the perturbation can be aligned to the magnetic field curvature effects (i.e., to the bumpiness) - since for toroidal axisymmetry the azimuthal variation is too large compared to the field variations.

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3.H. Grad, Proc. Nat. Acad. Sci., USA, 70, 3277 (1973)

Dupree

PF 15 334 (1972)

Lumps

THE IMPORTANCE OF ELECTROSTATIC TRAPPING OF HIGH-Z IMPURITY IONS IN TOKAMAK PLASMAS

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For a pure hydrogen plasma in the plateau regime neoclassical theory gives the electrostatic potential variation on a magnetic surface to be given by $e\Phi(\theta)/T \sim r\rho_i/RL$ where ρ_i is the proton Larmor radius in the poloidal field and L is the characteristic length for the ion temperature gradient. The fraction of high-Z impurity ions trapped in such a potential in existing Tokamak experiments would be large since typically $Z\rho_i/L >> 1$. The potential Φ has not been calculated for impure plasmas but the experimental measurements on the ST-Tokamak[1] show a large $\cos\theta$ potential variation for the constricted discharge studied. The trapping of the impurities leads to a strong θ-dependence of the main friction force experienced by the electrons and protons; when this friction has a $\cos\theta$ component, new neoclassical effects occur. Assuming most of the impurity ions are trapped and their density varies as $\cos\theta$, there is a new pinch effect involving passing particles with $\langle n v_r \rangle \sim -rE_\phi/RB_\theta$. This new term supplements the known neoclassical pinching and hence in the plateau regime causes a substantial increase. Secondly, an interaction with the Pfirsch-Schluter current leads to a reduction of the outward diffusion term. These effects could explain the enhanced pinching observed in the central core of ST.

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Guiding Center Plasma Effects in Helically Symmetric Equilibria

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Abstract

In order to study some collisionless plasma effects in plasma stability problems, the guiding center plasma model has been employed. Separate kinetic equations for ions and electrons are used and there is assumed to be an electric field parallel to the magnetic field which brings about charge neutrality. The equilibrium under consideration is assumed helically symmetric with helical wave number ε, $\varepsilon << 1$ and fractional distortion of the magnetic flux surfaces from cylinders δ, $\delta << 1$, $\varepsilon << \delta$. The ordering employed is the "new Scyllac" ordering. The distribution functions are assumed functions of the energy and the magnetic flux, and any other functional dependence is of order ε^2. The magnetic fields are then of the form

$$B_r = \varepsilon\delta a(r) \frac{\psi(r)}{r} \sin L(\theta-\varepsilon z) + \cdots .$$

$$B_\theta = \varepsilon\delta (a(r) \frac{\psi'(r)}{L} \cos L(\theta-\varepsilon z) + d(r)) + \cdots .$$

$$B_z = a(r) - \frac{\delta a'(r)\psi(r)}{rL} \cos L(\theta-\varepsilon z) + \cdots .$$

Thus, the screw pinch is included in the present analysis as a special case, $\psi(r) \equiv 0$.

The equations of motion are linearized and successively expanded in the parameters ε and then δ. Stability conditions analogous to fire hose, mirror, and velocity gradient conditions limit the class of stable distribution functions. Provided these conditions are met, one can show that the unstable modes and growth rates are the same as ideal non-dissipative magnetohydrodynamics. The stable modes will be affected by the presence of various stable continuous spectra peculiar to this kinetic problem. Thus, the range of applicability of ideal magnetohydrodynamics has been extended to this collisionless case. Further, the stabilization of higher m modes appears to lie outside guiding center theory.

NUMERICAL STUDIES OF NON-LINEAR EVOLUTION OF KINK MODES IN TOKOMAKS*

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ABSTRACT

A numerical method has been developed for following the non-linear behavior of kink modes in a Tokomak in the cylindrical approximation. We consider the case of zero resistivity. The plasma-vacuum interface is observed to undergo complicated elastic distortions and, for q near integral, to collapse inward to form one or more vacuum bubbles in the plasma interior. The range of q for which bubble formation is observed and the size of the bubble agree with predictions based on energy minimization.

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STUDIES OF THE WET WOOD BURNER

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Estimates of the efficiency of a two-component plasma as a reactor will be presented with emphasis on performing evaluations for plasma parameters and time scales anticipated in the next generation of toroidal confinement devices. Results to be presented center on time-dependent solutions of the multi-species Fokker-Planck equation including separate electron, deuteron, triton, and impurity species. In these calculations time dependent source and sink terms are being retained to model as accurately as possible the average behavior and energy balances of a real system. Thermonuclear reactions are included to provide an overall Q for the system to be compared with previous steady state "slowing-down" calculations of reactor efficiency. We wish to emphasize that minor species will be treated independently rather than including an average Z in the formalism and non-ideal losses which somewhat reduce the Q of the system will be included. As important suprathermal modifications to the Fokker-Planck coefficients are found they will be added to these time dependent calculations in a phenomenological manner.

$$\tau = \tau_{AC}\left(\langle E\,E\rangle\right)$$

$$= \frac{1}{k^2 D\left(1+\frac{B^2}{4\pi\rho c^2}\right)} \gg \frac{1}{k^2 D}$$

before $D = \sum_k \frac{\langle E_k^2\rangle c^2}{B^2}\tau(k)$

ind. of B for large B!

contradicts Taylor + McNamara, who got Bohm

something left out of Gr. C. (2D) theory: particle pairs (neutral pairs)

~~Ex~~ first $\longrightarrow$ across field

Pair diffusion: $D \propto \frac{1}{B}\ln B$

error to say $\tau \sim k^2 D$, since large $k \Rightarrow$ pairs appear

Very short wavelength ~~fluctuations~~ correlations not destroyed by particle ~~correlations~~ diffusion.

Ed Williams PhD thesis, D normann

IGNITION OF THERMONUCLEAR MICRO-EXPLOSIONS WITH INTENSE ION BEAMS.

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It is shown that due to their nonrelativistic velocities intense ion beams, generated in magnetically insulated diodes pulsed with a high voltage source(1), can be axially compressed increasing the beam power and shortening the beam pulse length by several orders of magnitude. It is furthermore shown that with this method an ion beam of a very high energy density can be produced by slowly building up the beam in about $\sim 10^{-6}$ sec, thus possibly permitting inductive energy storage devices to drive the beam. The axial beam compression is achieved by increasing the voltage at the diode during the pulse. In this way ions emitted at a later instance during the pulse have a higher velocity and catch up with ions emitted earlier resulting in the beam compression. In order to make this concept practical heavy ions have to be taken preferentially. The method can be also used to shape the time behaviour of the beam pulse to make possible high density compression of thermonuclear pellets by the action of several beams in a similar way as it had been proposed for laser fusion(2). In a similar way the same method can be also applied to ignite a micro fission or micro fission-fusion explosion by the compression of fissionable or hybrid pellets of fissionable and fusionable material(3,4).

1) F. Winterberg, Physics of High Energy Density, Academic Press, New York (1971) p. 397.
2) J. Nuckolls et al., Nature 239, 139 (1972).
3) F. Winterberg, Nature 241, 449 (1973).
4) F. Winterberg, in Laser Interaction and Related Plasma Phenomena, Vol. III, Plenum Press, New York (1974).

ON THE ELECTRON-CYCLOTRON MASER

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Borenstein and Lamb[1] have described a classical model of laser action in terms of the interaction of a beam of anharmonic oscillators with the electro-magnetic field in a resonant cavity. This classical model provides a basis for the description of the electron-cyclotron maser[1], a device which was previously suggested by Schneider[2] and first realized by Hirschfield and Wachtel[3]. The analysis of Borenstein, et al., has been extended and non-secular solutions of the equations of motion (with specific reference to the electron-cyclotron maser) have been obtained. If a semi-relativistic electron beam moving in a uniform magnetic field is used to pump-up the T.E. mode of an electromagnetic resonant cavity, the maximum amplitude attainable was estimated to be

$$\frac{eE}{m_e \Omega c} \sim \left(\frac{v_{o\perp}}{c} \right)^3,$$

where $v_{o\perp}$ is the initial perpendicular velocity of the electrons in the beam.

1) M. Borenstein and W.E. Lamb, Jr., Phys. Rev. A, 5, 1298 (1972).

2) J. Schneider, Phys. Rev. Lett. 2, 504 (1959).

3) J.L. Hirschfield and J.M. Wachtel, Phys. Rev. Lett. 12, 533 (1964).

Effect of Collisions on Impurity Drift Waves

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Investigations of the effect of collisions on the unstable drift waves associated with the presence of impurity ions have been made with a collision model of the Bhatnaghar-Gross-Krook type. The momentum exchange caused by impurity-hydrogen ion collisions is found to be the dominant mechanism in suitable parameter ranges, where it leads to a stabilizing effect. Included in the discussion will also be the effects of temperature gradient, curvature and shear.

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NOTES

Berk:
$$Cf = \nu\left[u\frac{\partial}{\partial u}\left[f(1+u^3)\right] + \frac{\alpha}{2}\frac{\partial}{\partial\mu}(1-\mu^2)\frac{\partial f}{\partial\mu}\right]$$

$$u = \frac{v}{v_\perp}, \quad v_\perp^3 = v_{the}\frac{m_e}{n_0}\frac{3\pi^{1/2}}{4}\sum_j n_j Z_j^2 / ?$$

$$\mu = \frac{v_z}{v}$$

Fokker-Planck Codes!

$\psi_r = 0 \Rightarrow q \equiv \frac{m}{n}$

NOTES

$B_z^2 >> B_\theta^2, p \Rightarrow kB_z \sim O(\frac{m}{r} B_\theta)$

and $\nabla_\perp \cdot V = 0 + O(\frac{B_\theta^2}{B_z^2})$

Ordering eliminates alfven + sound waves $B_{z1} \sim O(\frac{B_\theta}{B_{\theta 0}})$

Given helical sym $m\theta + kz$ $\frac{\partial}{\partial z} = \cdots \frac{\partial}{\partial \theta}$

$\frac{d\psi}{dt} = 0, \ \psi = A_z - \frac{kr}{m} A_\theta$

from ~~E~~ Maxwell.

$\frac{\partial \psi}{\partial t} + v \cdot \nabla \psi = 0 \quad (\eta = 0)$

$\nabla^2 \psi = j_z - 2\frac{kB_z}{m}$ $\quad \rho \frac{\partial V}{\partial t} = j \times B - \nabla p$

$= \nabla[\cdots]$

$- \nabla \psi \nabla^2 \psi$

$\nabla \cdot V = 0$

$p + \frac{B^2}{2}$ continuous, $\psi = \psi_w$ on wall

Callen: $\tau_E = \frac{\mu_0 R_0}{4\pi} \frac{\beta_p}{R\theta x}$ (profile function)

$\eta \sim \int d^3 \underline{v} \, \cancel{\oint} v_{\parallel} \, C_{eff}(f_1) \leftarrow ?$

Need impurity diffusion in PS regime